Minimum weighted clique cover on strip-composed perfect graphs

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Abstract. The only available combinatorial algorithm for the minimum weighted clique cover (MWCC) in claw-free perfect graphs is due to Hsu and Nemhauser [10] and dates back to 1984. More recently, Chudnovsky and Seymour [3] introduced a composition operation, strip-composition, in order to define their structural results for claw-free graphs; however, this composition operation is general and applies to non-claw-free graphs as well. In this paper, we show that a MWCC of a perfect strip-composed graph, with the basic graphs belonging to a class $\mathcal{G}$, can be found in polynomial time, provided that the MWCC problem can be solved on $\mathcal{G}$ in polynomial time. We also design a new, more efficient, combinatorial algorithm for the MWCC problem on strip-composed claw-free perfect graphs.

Keywords: claw-free graphs, perfect graphs, minimum weighted clique cover, odd pairs of cliques, strip-composed graphs.

1 Introduction

Given a graph G and a non-negative weight function w defined on the vertices of G, a *weighted clique cover* of G is a collection of cliques, with a non-negative weight y_C assigned to each clique C in the collection, such that, for each vertex v of G, the sum of the weights of the cliques containing v in the collection is at least $w(v)$. A *minimum weighted clique cover* of G (MWCC) is a clique cover such that the sum of the weights of all the cliques in the collection is minimum. When all weights are 1, a (minimum) weighted clique cover is simply called a *(minimum) clique cover*. It is known that for a perfect graph G, the weight $\tau_w(G)$ of a MWCC is the same as $\alpha_w(G)$, the weight of a *maximum weighted stable set* (MWSS) of G, that is, a set of pairwise nonadjacent vertices such that the sum of the weights of the vertices in the set is maximum.

In perfect graphs, the weight of a MWCC can be determined in polynomial time by using Lovász's $\theta_w(G)$ function. If one wants to compute also a MWCC

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of a perfect graph G (and not only the number $\tau_w(G)$), a polynomial time algorithm proposed by Grötschel, Lovász and Schrijver in [7] can be used. This algorithm is not combinatorial and it uses the $\theta_w(G)$ function combined with other techniques; however, for particular classes of perfect graphs, there also exist polynomial time combinatorial algorithms.

This is the case, for instance, for claw-free perfect graphs, where combinatorial algorithms for both the unweighted and the weighted version have been proposed by Hsu and Nemhauser [9, 10]. (A graph is *claw-free* if none of its vertices has a stable set of size three in its neighborhood.) The algorithm for the weighted case – in the paper we deal with this case, as it is more general – is essentially a "dual" algorithm as it relies on any algorithm for the MWSS problem in claw-free graphs (we have, nowadays, several algorithms for this, see [11, 12, 14, 5, 13, 15]), and, in fact, builds a MWCC by a clever use of linear programming complementarity slackness. The computational complexity of the algorithm by Hsu and Nemhauser is $O(|V(G)|^5)$. To the best of our knowledge, this is so far the only available combinatorial algorithm to solve the problem in claw-free perfect graphs.

In the last years a lot of efforts have been devoted to a better understanding of the structure of perfect graphs and of other relevant classes of graphs. Claw-free graphs in particular have been investigated, with an outstanding series of papers by Chudnovsky and Seymour (for a survey see [3]). The results by Chudnovsky and Seymour show that claw-free graphs with stability number greater than three are either *fuzzy circular interval graphs* (a generalization of *proper circular arc graphs*, we do not give the definition, as it is not interesting for this paper) or *strip-composed*, i.e., they are suitable composition of some basic graphs (the formal definition is given in the next section). Understanding this "2-case" structure of claw-free graphs has been the key for several developments for the MWSS problem [4, 14, 5] and the dominating set problem [8]. In particular, in [14] it is shown that a MWSS of a (non-necessarily claw-free) strip-composed graph, with the basic graphs belonging to a class $\mathcal{G}$, can be found in polynomial time by solving a matching problem, provided that the MWSS problem can be solved on $\mathcal{G}$ in polynomial time. Building upon this result, new algorithms for the MWSS problem in claw-free graphs are given in [14] and [5].

In this paper, we provide an analogous of the result in [14] for the MWCC problem. Namely, we show that a MWCC of a (non-necessarily claw-free) perfect strip-composed graph, with the basic graphs belonging to a class $\mathcal{G}$, can be found in polynomial time, provided that the MWCC problem can be solved on $\mathcal{G}$ in polynomial time. We point out that while the statement of this result goes along the same lines of the result in [14], its proof is by far more challenging. We apply this result to strip-composed claw-free perfect graphs, and provide a $O(|V(G)|^3)$-time algorithm for the MWCC problem that, differently from the $O(|V(G)|^5)$-time dual algorithm by Hsu and Nemhauser, has both a primal (on each basic graph we directly compute a MWCC) and a primal-dual flavour (on the composition of graphs we use a primal-dual algorithm for matching).

We shall consider finite, simple, loopless, undirected graphs. When dealing with multigraphs, we will say so explicitly. Let G be a graph. Denote by $V(G)$ its vertex set and by $E(G)$ its edge set. For a subset $V' \subseteq V(G)$, the *j-th neighborhood* $N_j(V')$ is the set of vertices $u \in V(G)$ at distance j from the set V'. When $V' = \{v\}$ we will write simply $N_j(v)$ and when $j = 1$ we will write just $N(V')$ (resp. $N(v)$). We will denote by $G[V']$ the subgraph of G induced by V', and by $G \setminus V'$ the subgraph of G induced by $V(G) \setminus V'$. Two sets $U, U' \subset V(G)$ are *complete* (*to each other*) if every vertex in U is adjacent to all the vertices in U'. They are *anticomplete* (*to each other*) if no vertex of U is adjacent to a vertex of U'.

A *claw* is a graph formed by a vertex with three neighbors of degree one. An *odd hole* is a chordless cycle of odd length at least 5. If H is a graph, a graph G is called *H-free* if no induced subgraph of G is isomorphic to H.

A graph is *cobipartite* if its vertex set can be covered by two cliques. A clique K of a connected graph G is *distance simplicial* if, for every j, its j-th neighborhood is also a clique. In this case, G is *distance simplicial* w.r.t. K (or simply distance simplicial). Note that a cobipartite graph is distance simplicial w.r.t. each of the two cliques covering its vertex set. Also it is not difficult to see that distance simplicial graphs are perfect.

The *intersection graph* of a family of sets $\mathcal{C}$ is the graph with vertex set $\mathcal{C}$, two sets in $\mathcal{C}$ being adjacent if and only if they intersect. The *line graph* $L(G)$ of a graph or multigraph G is the intersection graph of its edges. A graph H is a line graph if there is a graph or multigraph G such that $H = L(G)$ (G is called a *root graph* of H). A *star* or a *multistar* is the set of edges incident to a vertex v, while a *triangle* or *multitriangle* is a complete graph on three vertices with possibly multiple edges. A *matching* is a set of pairwise nonadjacent edges of a graph (two edges are adjacent if they share a vertex). Note that the multistars and multitriangles of a graph G correspond to the cliques of $L(G)$, while the matchings of G correspond to the stable sets of $L(G)$. Note also that the neighborhood of a vertex in a line graph can be always covered by two cliques. A graph is *quasi-line* if the neighborhood of each vertex is cobipartite. A quasi-line graph is, in particular, claw-free. Moreover, as observed by Hsu and Nemhauser in [10], a claw-free perfect graph is indeed quasi-line.

2 The MWSS problem on strip-composed perfect graphs

Chudnovsky and Seymour [3] introduced a composition operation in order to define their structural results for claw-free graphs. This composition operation is general and applies to non-claw-free graphs as well.

A *strip* $H = (G, \mathcal{A})$ is a graph G (not necessarily connected) with a multi-family $\mathcal{A}$ of either one or two designated non-empty cliques of G. The cliques in $\mathcal{A}$ are called the *extremities* of H, and H is said a *1-strip* if $|\mathcal{A}| = 1$, and a *2-strip* if $|\mathcal{A}| = 2$. Let $\mathcal{G} = (G^1, \mathcal{A}^1), \ldots, (G^k, \mathcal{A}^k)$ be a family of k vertex disjoint strips, and let $\mathcal{P}$ be a partition of the multi-set of the cliques in $\mathcal{A}^1 \cup \ldots \cup \mathcal{A}^k$. The *composition* of the k strips w.r.t. $\mathcal{P}$ is the graph G that is obtained from the union of $G^1, \dots, G^k$, by making adjacent vertices of $A \in \mathcal{A}^i$ and $B \in \mathcal{A}^j$ (i, j not necessarily different) if and only if A and B are in the same class of the partition $\mathcal{P}$. In this case we also say that $(\mathcal{G}, \mathcal{P})$, where $\mathcal{G} = \{(G^j, \mathcal{A}^j), j \in 1, \dots, k\}$, defines a *strip decomposition* of G. Note that we can assume w.l.o.g. that each graph G^i is an induced subgraph of G.

We say that a graph G is *strip-composed* if G is a composition of some set of strips w.r.t. some partition $\mathcal{P}$. Each class of the partition of the extremities $\mathcal{P}$ defines a clique of the composed graph, and is called a *partition-clique*. We denote the extremities of the strip H_i by $\mathcal{A}^i = \{A^i_1, A^i_2\}$ if H_i is a 2-strip and by $\mathcal{A}^i = \{A^i_1\}$ if H_i is a 1-strip. We often abuse notations, and when we refer to a vertex of a strip (or to a stable set of a strip etc.) we indeed consider a vertex (or a stable set etc.) of the graph in the strip.

The composition operation preserves some graph properties. Given a 2-strip $(G, \{A_1, A_2\})$, the graph G_+ is obtained from G by adding two vertices a_1, a_2 such that $N(a_j) = A_j$, for $j = 1, 2$; for a 1-strip $(G, \{A_1\})$ the graph G_+ is obtained from G by adding a vertex a_1 such that $N(a_1) = A_1$. A strip $(G, \mathcal{A})$ is claw-free/quasi-line/line if the graph G_+ is claw-free/quasi-line/line. The composition of claw-free/quasi-line/line strips is a claw-free/quasi-line/line graph (see e.g. [5]).

Suppose we are given a graph G and its strip decomposition $(\mathcal{G}, \mathcal{P})$. In [14] it is shown how to exploit this decomposition in order to solve the MWSS on G.

Theorem 1. *[14] Let G be the composition of strips $H_i = (G^i, \mathcal{A}^i)$ $i = 1, \dots, k$ w.r.t. a partition $\mathcal{P}$. Suppose that for each $i = 1, \dots, k$ one can compute a* MWSS *of H_i in time $O(p_i(|V(G^i)|))$. Then the* MWSS *problem on G can be solved in time $O(\sum_{i=1}^k p_i(|V(G^i)|) + match(|V(G)|))$, where $match(n)$ is the time required to solve the matching problem on a graph with n vertices. If $p_i(|V(G^i)|)$ is polynomial for each i, then the* MWSS *can be solved on G in polynomial time.*

In order to prove their theorem [14], the authors replace every strip H_i with a suitable simpler *gadget strip* T_i, that is a single vertex for each 1-strip and a triangle for each 2-strip (in this second case the extremities are two different edges of the triangle). Then they define a weight function on the vertices of those simpler strips; for every strip H_i with extremities A^i_1 and A^i_2 this function depends on the values $\alpha_w(G^i)$, $\alpha_w(G^i \setminus A^i_1)$, $\alpha_w(G^i \setminus A^i_2)$, $\alpha_w(G^i \setminus (A^i_1 \cup A^i_2))$ and $\alpha_w(G^i \setminus (A^i_1 \Delta A^i_2))$. Thus, if one can compute a MWSS of G^i in polynomial time, then one can compute the weight function of the simpler strips in polynomial time.

They define a suitable partition $\tilde{\mathcal{P}}$ of the extremities of the gadget strips. In this way they obtain a graph $\tilde{G}$ which is the strip-composition of the strips T_i, $i = 1, \dots, k$, w.r.t. the partition $\tilde{\mathcal{P}}$, and, since the strips are line strips, this graph is line. Moreover, from the construction of the simpler strips and of the weights, it is easy to translate a MWSS of $\tilde{G}$ into a MWSS of G. Finally, as $\tilde{G}$ is a line-graph, they can find a MWSS of $\tilde{G}$ by building the root graph of $\tilde{G}$ and computing a maximum weighted matching in this graph.

3 The MWCC problem on strip-composed perfect graphs

Suppose we are given a perfect graph G that is the composition of strips $H_1 = (G^1, \mathcal{A}^1), \ldots, H_k = (G^k, \mathcal{A}^k)$ w.r.t. a partition $\mathcal{P}$, and a non-negative weight function w on $V(G)$. In this section we will show how to exploit the decomposition in order to solve the MWCC on G. We will follow the approach outlined in the previous section for the MWSS; however, as we explain in the following, the task is now much more challenging.

We will compute a MWCC of G in three steps. **Step 1.** We replace each strip H_i by a simple *gadget strip* $\tilde{H}_i = (\tilde{G}^i, \tilde{\mathcal{A}}^i)$ and compose the strips $\tilde{H}_i$ with respect to a suitable partition of the multi-set $\bigcup_{i=1..k} \tilde{\mathcal{A}}^i$ so as to obtain a graph $\tilde{G}$. However, we cannot use the gadget strips defined in the previous section, as the graph $\tilde{G}$ might be imperfect: this will lead us to define four different new gadgets, with different parity properties, that are such that $\tilde{G}$ is odd hole free and line, thus perfect [16]. We also define a suitable weight function $\tilde{w}$ on the vertices of $\tilde{G}$, as well as new weight functions $w^1, \ldots, w^k$ on the vertices of each strip. **Step 2.** Following [16], we may find a MWCC of $\tilde{G}$, w.r.t. the weight $\tilde{w}$, by running a primal-dual algorithm for the maximum weighted matching [6] on the root graph of $\tilde{G}$. **Step 3.** We reconstruct a MWCC of G from a MWCC of $\tilde{G}$ and a MWCC of each of the strips H_i w.r.t. the weight function w^i. Again, this will be more involved than for the MWSS problem, because unfortunately there is not always a direct correspondence between cliques of $\tilde{G}$ and cliques of G. Moreover, for some 2-strips $H_i = (G^i, \mathcal{A}^i)$, besides a MWCC of the strip, we will also need to compute a MWCC of some auxiliary graphs associated to the strip: the graph $G^i_{\bullet}$ that is obtained from G^i by adding a new vertex x complete to both A^i_1 and A^i_2 and the graph $G^i_{=}$ that is the graph obtained from G^i by making A^i_1 complete to A^i_2.

In order to give a few more details we need some additional definitions. Let $U, W \subseteq V(G)$. We call a path $P = v_1, \ldots, v_k$ ($k \geq 1$) a *U–W path* if P is chordless, $v_1 \in U$, $v_k \in W$, and $v_i \notin U \cup W$ for $2 \leq i \leq k-1$. A 2-strip $H_i = (G^i, \mathcal{A}^1 = \{A^i_1, A^i_2\})$ will be called *non-connected* if there is no A^i_1–A^i_2 path, and *connected* otherwise. We say that a connected 2-strip H_i is *even* (resp. *odd*) if every A^i_1–A^i_2 path has even (resp. odd) length. If a connected 2-strip has both even and odd length A^i_1–A^i_2 paths, then we say that H_i is an *even-odd* strip. We call an odd or even-odd strip H_i *odd-short* if every odd A^i_1–A^i_2 path has length one, and we call an even or even-odd strip H_i *even-short* if every even A^i_1–A^i_2 path has length zero (i.e., it consists of a vertex in $A^i_1 \cap A^i_2$). (With the notation of [1], H_i is an odd strip if and only if A^i_1 and A^i_2 are an odd pair of cliques in G^i.)

Theorem 2. *Let G be a perfect graph, composition of strips $H_i = (G^i, \mathcal{A}^i)$ $i = 1, \ldots, k$ w.r.t. a partition $\mathcal{P}$. For each $i = 1, \ldots, k$ let $O(p_i(|V(G^i)|))$ be the time required to compute:*

- *a MWCC of G^i and of $G^i_{\bullet}$, if H_i is an odd-short strip and $G^i_{\bullet}$ is an induced subgraph of G (thus perfect);*

– *a* MWCC *of G^i and of $G^i_=$, if $G^i_=$ is an induced subgraph of G (thus perfect), A^i_1 and in A^i_2 belong to the same class of $\mathcal{P}$, and there is an A_1–A_2 path of length two in the strip. In this case, when solving the* MWCC *on $G^i_=$, one can restrict to the case where the weight function w^i defined on $V(G^i_=)$ is such that $\alpha_{w^i}(G^i_=) = \alpha_{w^i}(G^i_= \setminus (A^i_1 \cup A^i_2))$;*
– *a* MWCC *of G^i else.*

Then the MWCC *problem on G can be solved in time $O(\sum_{i=1}^k p_i(|V(G^i)|) + match(|V(G)|))$, where $match(n)$ is the time required to solve the matching problem on a graph with n vertices. If $p_i(|V(G^i)|)$ is polynomial for each i, then the* MWCC *can be solved on G in polynomial time.*

We devote the rest of this section to provide more details about Theorem 2 and its proof. We first deal with the gadget strips (that in this section we simply call gadgets) that will compose the graph $\tilde{G}$ and establish the relation between $\tau_w(G)$ and $\tau_{\tilde{w}}(\tilde{G})$. We make a heavy use of duality between the MWCC and the MWSS problem: the fact that for every induced subgraph J of G, $\alpha_w(J) = \tau_w(J)$, is due to the perfection of G. We use this relation to easily prove the correctness of the weight function defined on the vertices of each gadget.

To design the gadgets, we delve into three cases: (i) $H_i = (G^i, \mathcal{A}^i)$ is a 1-strip; (ii) $H_i = (G^i, \mathcal{A}^i)$ is a 2-strip with the extremities in the same class of the partition $\mathcal{P}$; (iii) $H_i = (G^i, \mathcal{A}^i)$ is a 2-strip with the extremities in different classes of the partition.

$(i)-(ii)$ For the first two cases the gadget will be a single vertex. In particular we define the trivial 1-strip $\tilde{H}^0_i = (T^i_0, \tilde{\mathcal{A}}^i_0)$, where the graph T^i_0 consists of a single vertex c^i, and $\tilde{\mathcal{A}}^i_0 = \{\{c^i\}\}$. Moreover, for (i) we let $\delta^i_1 = \alpha_w(G^i \setminus A^i_1)$ and define $\tilde{w}(c^i) = \alpha_w(G^i) - \delta^i_1$. For (ii) we let $\delta^i_1 = \alpha_w(G^i \setminus (A^i_1 \cup A^i_2))$ and define $\tilde{w}(c^i) = \max\{\alpha_w(G^i \setminus A^i_1), \alpha_w(G^i \setminus A^i_2), \alpha_w(G^i \setminus (A^i_1 \triangle A^i_2))\} - \delta^i_1$. Finally, if we use $\tilde{H}^0_i$ instead of H_i in the composition, the new partition is $\mathcal{P}' := (\mathcal{P} \setminus \{P\}) \cup \{(P \setminus \mathcal{A}^i) \cup \tilde{\mathcal{A}}^i\}$, where $P \in \mathcal{P}$ was the set containing $\mathcal{A}^i$. We can show that replacing a 1-strip or a 2-strip with both extremities in the same class of $\mathcal{P}$ by its corresponding gadget strip makes the value of the MWSS drop by δ^i_1.

(iii) Let us consider a 2-strip $H_i = (G^i, \mathcal{A}^i)$ with the extremities in different classes of the partition $\mathcal{P}$. We want to introduce a gadget $\tilde{H}_i = (\tilde{G}^i, \tilde{\mathcal{A}}^i)$ and a new weight function $\tilde{w}$ on the vertices of $\tilde{G}^i$ in such a way that, when replacing H_i by $\tilde{H}_i$ in the strip-composition for a suitable partition, the difference between the weights of the MWSS of the original graph and the MWSS of the new graph is δ^i_1, where $\delta^i_1 = \alpha_w(G^i \setminus (A^i_1 \cup A^i_2))$.

This is satisfied by the gadget defined in [14], but it is an even-odd strip, and we need to take into consideration the parity of the strips, otherwise the composition may introduce odd holes. We will introduce three gadgets (an odd strip, an even strip and a non-connected one). None of them will work for all the cases, but depending on the fact that the relation $\alpha_w(G^i \setminus A^i_1) + \alpha_w(G^i \setminus A^i_2) \gtreqless \alpha_w(G^i) + \delta^i_1$ is satisfied with $=$, $>$ or $<$. We will see later on that the satisfaction of this relation is strictly related to the parity of the strips. Given a 2-strip $H_i = (G^i, \mathcal{A}^i)$, we define three associated trivial strips as follows:

(a) $\tilde{H}_i^1 = (T_1^i, \tilde{\mathcal{A}}_1^i)$ such that $V(T_1^i) = \{u_1^i, u_2^i\}$, $E(T_1^i) = \emptyset$, $\tilde{\mathcal{A}}_1^i = \{\tilde{A}_1^i, \tilde{A}_2^i\}$ and $\tilde{A}_1^i = \{u_1^i\}$, $\tilde{A}_2^i = \{u_2^i\}$. The new weight function $\tilde{w}$ gives the following weights to the vertices of T_1^i: $\tilde{w}(u_1^i) = \alpha_w(G^i \setminus A_2^i) - \delta_1^i$, $\tilde{w}(u_2^i) = \alpha_w(G^i \setminus A_1^i) - \delta_1^i$.

(b) $\tilde{H}_i^2 = (T_2^i, \tilde{\mathcal{A}}_2^i)$ such that $V(T_2^i) = \{u_1^i, u_2^i, u_3^i\}$, $E(T_2^i) = \{u_1^i u_2^i, u_2^i u_3^i\}$, $\tilde{\mathcal{A}}_2^i = \{\tilde{A}_1^i, \tilde{A}_2^i\}$ and $\tilde{A}_1^i = \{u_1^i, u_2^i\}$, $\tilde{A}_2^i = \{u_3^i\}$. The new weight function $\tilde{w}$ gives the following weights to the vertices of T_2^i: $\tilde{w}(u_1^i) = \alpha_w(G^i) - \alpha_w(G^i \setminus A_1^i)$, $\tilde{w}(u_2^i) = \alpha_w(G^i \setminus A_2^i) - \delta_1^i$, $\tilde{w}(u_3^i) = \alpha_w(G^i \setminus A_1^i) - \delta_1^i$.

(c) $\tilde{H}_i^3 = (T_3^i, \tilde{\mathcal{A}}_3^i)$ such that $V(T_3^i) = \{u_1^i, u_2^i, u_3^i\}$, $E(T_3^i) = \{u_1^i u_2^i, u_2^i u_3^i\}$, $\tilde{\mathcal{A}}_3^i = \{\tilde{A}_1^i, \tilde{A}_2^i\}$ and $\tilde{A}_1^i = \{u_1^i, u_2^i\}$, $\tilde{A}_2^i = \{u_2^i, u_3^i\}$. The new weight function $\tilde{w}$ gives the following weights to the vertices of T_3^i: $\tilde{w}(u_1^i) = \alpha_w(G^i \setminus A_2^i) - \delta_1^i$, $\tilde{w}(u_2^i) = \alpha_w(G^i) - \delta_1^i$, $\tilde{w}(u_3^i) = \alpha_w(G^i \setminus A_1^i) - \delta_1^i$.

If we use either $\tilde{H}_i^1$, $\tilde{H}_i^2$ or $\tilde{H}_i^3$ instead of H_i in the composition, the new partition is $\mathcal{P}' := \mathcal{P} \setminus \{P_1, P_2\} \cup \{(P_1 \setminus \{A_1^i\}) \cup \{\tilde{A}_1^i\}, (P_2 \setminus \{A_2^i\}) \cup \{\tilde{A}_2^i\}\}$, where $P_1, P_2 \in \mathcal{P} : A_1^i \in P_1, A_2^i \in P_2$.

Lemma 1. *Let G be the composition of strips $H_1 = (G^1, \mathcal{A}^1), \dots, H_k = (G^k, \mathcal{A}^k)$ w.r.t. a partition $\mathcal{P}$, and let w be a non-negative weight function defined on the vertices of G. Suppose that H_1 is a 2-strip with the extremities in different classes of the partition $\mathcal{P}$. For some $j \in \{1, 2, 3\}$, let G' be the composition of strips $\tilde{H}_1^j = (T_j^1, \mathcal{A}_j^1), H_2 = (G^2, \mathcal{A}^2), \dots, H_k = (G^k, \mathcal{A}^k)$ w.r.t. the partition $\mathcal{P}'$ previously defined. Let w' be the weight function defined on the vertices of G' as $w'(v) = w(v)$ for $v \in \bigcup_{i=2..k} V(G^i)$, and $w'(v) = \tilde{w}(v)$ for $v \in V(T_j^1)$.*

(a) If $j = 1$ and $\alpha_w(G^1 \setminus A_1^1) + \alpha_w(G^1 \setminus A_2^1) = \alpha_w(G^1) + \delta_1^1$, then $\alpha_w(G) = \alpha_{w'}(G') + \delta_1^1$.

(b) If $j = 2$ and $\alpha_w(G^1 \setminus A_1^1) + \alpha_w(G^1 \setminus A_2^1) \geq \alpha_w(G^1) + \delta_1^1$, then $\alpha_w(G) = \alpha_{w'}(G') + \delta_1^1$.

(c) If $j = 3$ and $\alpha_w(G^1 \setminus A_1^1) + \alpha_w(G^1 \setminus A_2^1) \leq \alpha_w(G^1) + \delta_1^1$, then $\alpha_w(G) = \alpha_{w'}(G') + \delta_1^1$.

Lemma 2. *The following relations hold depending of the connection and parity of a 2-strip $H_1 = (G^1, \mathcal{A}^1)$:*

(a) if it is non-connected then $\alpha_w(G^1 \setminus A_1^1) + \alpha_w(G^1 \setminus A_2^1) = \alpha_w(G^1) + \delta_1^1$;

(b) if it is odd and G^1 perfect then $\alpha_w(G^1 \setminus A_1^1) + \alpha_w(G^1 \setminus A_2^1) \geq \alpha_w(G^1) + \delta_1^1$;

(c) if it is even and G^1 perfect then $\alpha_w(G^1 \setminus A_1^1) + \alpha_w(G^1 \setminus A_2^1) \leq \alpha_w(G^1) + \delta_1^1$.

We now give a method to choose one gadget for every 2-strip H_i. If we can calculate the values of the minimum weighted clique covers $\tau_w(G^i)$, $\tau_w(G^i \setminus A_2^i)$, $\tau_w(G^i \setminus A_1^i)$ and $\tau_w(G^i \setminus (A_1^i \cup A_2^i))$ for each strip, we can determine which one of these three relations holds

1. $\tau_w(G^i \setminus A_1^i) + \tau_w(G^i \setminus A_2^i) = \tau_w(G^i) + \tau_w(G^i \setminus (A_1^i \cup A_2^i))$
2. $\tau_w(G^i \setminus A_1^i) + \tau_w(G^i \setminus A_2^i) > \tau_w(G^i) + \tau_w(G^i \setminus (A_1^i \cup A_2^i))$
3. $\tau_w(G^i \setminus A_1^i) + \tau_w(G^i \setminus A_2^i) < \tau_w(G^i) + \tau_w(G^i \setminus (A_1^i \cup A_2^i))$

If 1) holds we can simply use $\tilde{H}_i^1$ as a suitable gadget. If 2) holds we know that the strip is either odd or even-odd and we can use $\tilde{H}_i^2$ as a suitable gadget. If 3) holds we know that the strip is either even or even-odd and we can use $\tilde{H}_i^3$ as a suitable gadget.

Remark 1. Let G be the composition of the strips $H_1, H_2, \dots, H_k$ with respect to a partition $\mathcal{P}$ and suppose that G is odd hole free. Let G' be the composition of $\tilde{H}_1^j, H_2, \dots, H_k$ with respect to the partition $\mathcal{P}'$ previously defined. For $j = 0, 1$, G' is odd hole free. If H_1 is odd or even-odd and $j = 2$, then G' is odd hole free. If H_1 is even or even-odd and $j = 3$, then G' is odd hole free.

Strips $\tilde{H}_i^0, \tilde{H}_i^1, \tilde{H}_i^2, \tilde{H}_i^3$ are line strips. So, if we iteratively replace each strip H_i by the suitable gadget $\tilde{H}_i^j$, according to the validity of 1, 2 or 3, the graph $\tilde{G}$ we obtain is odd hole free and a line graph, thus perfect [16]. As a corollary of the previous Lemmas, it follows that $\alpha_w(G) = \alpha_{\tilde{w}}(\tilde{G}) + \sum_{i=1}^{k} \delta_1^i$. Since both graphs are perfect, by duality the same relation holds for the values of the MWCC of the two graphs.

$\tilde{G}$ is a line, perfect graph. Let H be a multigraph that is a root of $\tilde{G}$. Following [16], we may build a MWCC of $\tilde{G}$, by a primal-dual algorithm for the maximum weighted matching [6]: this is because each maximal clique of $\tilde{G}$ corresponds to either a multistar of H or to a multitriangle of H. We therefore compute a MWCC of $\tilde{G}$, w.r.t. the weight $\tilde{w}$. We now need to "translate" this into a MWCC of G, w.r.t. the weight w. However, there is a catch: unfortunately there are some cliques of $\tilde{G}$ that do not correspond to any clique of G. In order to deal with this problem, we detail the structure of H.

Remark 2. Suppose that $\mathcal{P} = \{P_1, \dots, P_r\}$. Then H is composed by: a set of vertices $\{x_1, \dots, x_r\}$, each x_i corresponding to the class P_i of $\mathcal{P}$; an edge $x_j x_\ell$ for each strip H_i such that we use $\tilde{H}_i^3$ in the composition and such that $A_1^i \in P_j$ and $A_2^i \in P_\ell$ (this edge corresponds to the vertex u_2^i of T_3^i); vertices z_j^i and z_ℓ^i and edges $z_j^i x_j$ and $z_\ell^i x_\ell$ for each strip H_i such that we use $\tilde{H}_i^3$ in the composition and such that $A_1^i \in P_j$ and $A_2^i \in P_\ell$ (the edges $z_j^i x_j$ and $z_\ell^i x_\ell$ correspond to the vertices u_1^i and u_3^i of T_3^i, respectively); a vertex $y_{j\ell}^i$ and edges $y_{j\ell}^i x_j$ and $y_{j\ell}^i x_\ell$ for each strip H_i such that we use $\tilde{H}_i^2$ in the composition and such that $A_1^i \in P_j$ and $A_2^i \in P_\ell$ (the edges $y_{j\ell}^i x_j$ and $y_{j\ell}^i x_\ell$ correspond to the vertices u_2^i and u_3^i of T_2^i, respectively); a vertex z_j^i and an edge $z_j^i x_j$ for each strip H_i such that we use $\tilde{H}_i^2$ in the composition and such that $A_1^i \in P_j$ (the edge corresponds to the vertex u_1^i of T_2^i); a vertex z_j^i and an edge $z_j^i x_j$ for each strip H_i such that we use $\tilde{H}_i^0$ in the composition and such that $A_1^i \in P_j$ (the edge corresponds to the vertex c^i of T_0^i); vertices z_j^i and z_ℓ^i and edges $z_j^i x_j$ and $z_\ell^i x_\ell$ for each strip H_i such that we use $\tilde{H}_i^1$ in the composition and such that $A_1^i \in P_j$ and $A_2^i \in P_\ell$ (the edges $z_j^i x_j$ and $z_\ell^i x_\ell$ correspond to the vertices u_1^i and u_2^i of T_1^i, respectively).

The maximal cliques of $\tilde{G}$ correspond to the multistars and multitriangles of H, i.e., the multistars centered at x_j for $j = 1, \dots, r$, the possible multitriangles $x_i x_j x_\ell$ for i, j, ℓ pairwise distinct elements in $\{1, \dots, r\}$, and, for each vertex $y_{j\ell}^i$, either the star centered at $y_{j\ell}^i$ or the multitriangle $y_{j\ell}^i x_j x_\ell$ with $j, \ell \in \{1, \dots, r\}$ and $j \neq \ell$, depending on the existence of edges joining x_j and x_ℓ. To each of these cliques of $\tilde{G}$ we will assign a clique of G, except for the case of cliques involving y-vertices. We have to deal with those cliques in a different way.

To the clique of $\tilde{G}$ corresponding to the multistar centered at x_j in H, we will assign in G the partition-clique $\bigcup_{A^i_d \in P_j} A^i_d$. To the clique of $\tilde{G}$ corresponding to the multitriangle $x_i x_j x_\ell$ in H, we will assign in G the clique induced by $\bigcup_{d \in I_{ij\ell}} (A^d_1 \cap A^d_2)$, where $I_{ij\ell}$ is the set of indices d of 2-strips in the decomposition, that have been replaced by $\tilde{H}^3_d$, and having their two extremities belonging to two different sets in $\{P_i, P_j, P_\ell\}$ (we can prove that these intersections are nonempty).

Now we want to show how we deal with the star centered at $y^i_{j\ell}$ and the multitriangles $y^i_{j\ell} x_j x_\ell$. As we have already said, these two structures correspond to cliques in $\tilde{G}$, but the corresponding cliques in $\tilde{G}$ cannot be extended to cliques of G. Thus we have to show that we can rearrange the weight function of the vertices of the strips in order to get a cover with the same value which includes only cliques. First we show that if we have a multitriangle $y^i_{j\ell} x_j x_\ell$ in H, then the 2-strip $(G^i, \mathcal{A}^i)$ is odd-short, and there is a vertex x complete to both extremities of it in G. Then we prove the following lemma. This lemma essentially says that if we have assigned a weight $a > 0$ to the triangle $y^i_{j\ell} x_j x_\ell$ then we can discard this triangle and ask for a MWCC of value $\delta^i_1 + a$ in the graph induced by G^i and $A^k_1 \cap A^k_2$ for every k such that $A^k_1 \cap A^k_2$ is complete to $A^i_1 \cup A^i_2$ (this set of vertices form a clique), in such a way that $\bigcup_k (A^k_1 \cap A^k_2)$ is covered by a quantity greater or equal to a. W.l.o.g. we may consider that we need to cover just an extra vertex x of weight a complete to $A^i_1 \cup A^i_2$.

Lemma 3. *Let $H_i = (G^i, \mathcal{A}^i)$ be a 2-strip. Let $G^i_\bullet$ be the graph obtained from G^i by adding a new vertex x complete to both A^i_1 and A^i_2. Let w be a non-negative weight function defined on the vertices of G^i. Let $\delta^i_1 = \alpha_w(G^i \setminus (A^i_1 \cup A^i_2))$. Let a, b_1, b_2 be non-negative numbers such that $b_1 \geq \alpha_w(G^i) - \alpha_w(G^i \setminus A^i_1)$, $a + b_1 \geq \alpha_w(G^i \setminus A^i_2) - \delta^i_1$, $a + b_2 \geq \alpha_w(G^i \setminus A^i_1) - \delta^i_1$, and let w^i be defined as $w^i(v) = w(v)$ for $v \in V(G^i) \setminus (A^i_1 \cup A^i_2)$, $w^i(v) = \max\{0, w(v) - b_1\}$ for $v \in A^i_1 \setminus A^i_2$, $w^i(v) = \max\{0, w(v) - b_2\}$ for $v \in A^i_2 \setminus A^i_1$, and $w^i(v) = \max\{0, w(v) - b_1 - b_2\}$ for $v \in A^i_1 \cap A^i_2$. Then $\alpha_{w^i}(G^i_\bullet) = \delta^i_1 + a$. In particular, $\alpha_{w^i}(G^i) \leq \delta^i_1 + a$.*

We underline that the last sentence of Lemma 3 suggests also how to "translate" the weight a possibly assigned to the star centered in $y^i_{j\ell}$ and, in general, how to deal with the strips that have been replaced by $\tilde{H}^2$.

Now we want to show that if we have a weighted clique cover of $\tilde{G}$, we can cover the "residual" weight w^i of each strip $H_i = (G^i, \mathcal{A}^i)$ with a weighted clique cover of value at most δ^i. The following lemmas give the desired result for 1-strips and 2-strips that have been replaced with $\tilde{H}^1$ or $\tilde{H}^3$. In particular, Lemma 6 considers the case of 2-strips with a non empty intersection of the extremities that might cause multitriangles in the root graph of $\tilde{G}$.

Lemma 4. *Let $H_i = (G^i, \mathcal{A}^i)$ be a 1-strip and let w be a non-negative weight function defined on the vertices of G^i. Let $\delta^i_1 = \alpha_w(G^i \setminus A^i_1)$, let $b \geq \alpha_w(G^i) - \delta^i_1$, and let w^i be defined as $w^i(v) = w(v)$ for $v \in V(G^i) \setminus A^i_1$, $w(v) = \max\{0, w(v) - b\}$ for $v \in A^i_1$. Then $\alpha_{w^i}(G^i) = \delta^i_1$.*

Lemma 5. *Let $H_i = (G^i, \mathcal{A}^i)$ be a 2-strip and let w be a non-negative weight function defined on the vertices of G^i. Let $\delta_1^i = \alpha_w(G^i \setminus (A_1^i \cup A_2^i))$. Let b_1, b_2 be numbers such that $b_1 \geq \alpha_w(G^i \setminus A_2^i) - \delta_1^i$, $b_2 \geq \alpha_w(G^i \setminus A_1^i) - \delta_1^i$, and $b_1 + b_2 \geq \alpha_w(G^i) - \delta_1^i$, and let w^i be defined as $w^i(v) = w(v)$ for $v \in V(G^i) \setminus (A_1^i \cup A_2^i)$, $w^i(v) = \max\{0, w(v) - b_1\}$ for $v \in A_1^i \setminus A_2^i$, $w^i(v) = \max\{0, w(v) - b_2\}$ for $v \in A_2^i \setminus A_1^i$, and $w^i(v) = \max\{0, w(v) - b_1 - b_2\}$ for $v \in A_1^i \cap A_2^i$. Then $\alpha_{w^i}(G^i) = \delta_1^i$.*

Lemma 6. *Let $H_i = (G^i, \mathcal{A}^i)$ be an even-short 2-strip such that G^i is perfect, and let w be a non-negative weight function defined on the vertices of G^i. Let $\delta_1^i = \alpha_w(G^i \setminus (A_1^i \cup A_2^i))$. Let b_1, b_2, a be numbers such that $b_1 \geq \alpha_w(G^i \setminus A_2^i) - \delta_1^i$, $b_2 \geq \alpha_w(G^i \setminus A_1^i) - \delta_1^i$, and $a + b_1 + b_2 \geq \alpha_w(G^i) - \delta_1^i$, and let w^i be defined as $w^i(v) = w(v)$ for $v \in V(G^i) \setminus (A_1^i \cup A_2^i)$, $w^i(v) = \max\{0, w(v) - b_1\}$ for $v \in A_1^i \setminus A_2^i$, $w^i(v) = \max\{0, w(v) - b_2\}$ for $v \in A_2^i \setminus A_1^i$, and $w^i(v) = \max\{0, w(v) - b_1 - b_2 - a\}$ for $v \in A_1^i \cap A_2^i$. Then $\alpha_{w^i}(G^i) = \delta_1^i$.*

Finally, we analyze the case of 2-strips with both extremities in the same class of $\mathcal{P}$. Such a strip H_i has been replaced with $\tilde{H}_i^0$, thus every vertex in its extremities is covered by a quantity of at least $\max\{\alpha_w(G^i \setminus A_1^i), \alpha_w(G^i \setminus A_2^i), \alpha_w(G^i \setminus (A_1^i \triangle A_2^i))\} - \delta_1^i$.

Lemma 7. *Let $H_i = (G^i, \mathcal{A}^i)$ be a 2-strip. Let $G^i_=$ be the graph obtained from G^i by adding the edges between A_1^i and A_2^i. Let w be a non-negative weight function defined on the vertices of G^i. Let $\delta_1^i = \alpha_w(G^i \setminus (A_1^i \cup A_2^i))$, let $b \geq \max\{\alpha_w(G^i \setminus A_1^i), \alpha_w(G^i \setminus A_2^i), \alpha_w(G^i \setminus (A_1^i \triangle A_2^i))\} - \delta_1^i$, and let w^i be defined as $w^i(v) = w(v)$ for $v \in V(G^i) \setminus (A_1^i \cup A_2^i)$, $w(v) = \max\{0, w(v) - b\}$ for $v \in A_1^i \cup A_2^i$. Then $\alpha_{w^i}(G^i_=) = \delta_1^i$. Moreover, if $G^i_=$ is perfect, any* MWCC *of $G^i_=$ w.r.t. w^i does not assign strictly positive weight to the clique $A_1^i \cup A_2^i$.*

Note that the last sentence of the previous lemma implies that, if $G^i_=$ is perfect and there are no two vertices $v_1 \in A_1^i$ and $v_2 \in A_2^i$ having a common neighbor in $V(G^i) \setminus (A_1^i \cup A_2^i)$, then any MWCC of $G^i_=$ w.r.t. w^i is in fact a MWCC of G^i w.r.t. w^i. We also observe that whenever we cannot use Lemma 7 we must be able to compute a MWCC of $G^i_=$ in order to reconstruct a clique cover of G from a clique cover of $\tilde{G}$. This is why we require in Theorem 2 that a MWCC of $G^i_=$ can be computed in time $O(p_i(|V(G^i)|))$ in that case.

4 Application to strip-composed claw-free perfect graphs

As an application of Theorem 2, we give a new algorithm for the MWCC on strip-composed claw-free perfect graphs. Recall that claw-free perfect graphs are in fact quasi-line. In the last decade the structure of quasi-line graphs was deeply investigated, with some results providing a detailed description and characterization of the strips that, through composition, can be part of a quasi-line graph. This is the case of the structure theorem by Chudnovsky and Seymour in [2]. The following algorithmic decomposition theorem from [5] applies to quasi-line graphs. (A *net* is a graph formed by a triangle and three vertices of degree one, each of them adjacent to a distinct vertex of the triangle.)

Theorem 3. *[5] Let G be a connected quasi-line graph. In time $O(|V(G)||E(G)|)$, one can either recognize that G is net-free; or provide a decomposition of G into $k \leq |V(G)|$ quasi-line strips $(G^1, \mathcal{A}^1), \ldots, (G^k, \mathcal{A}^k)$, w.r.t. a partition $\mathcal{P}$, such that each graph G^i is distance simplicial w.r.t. each clique $A \in \mathcal{A}^i$. Moreover, if $\mathcal{A}^i = \{A^i_1, A^i_2\}$, then either $A^i_1 = A^i_2 = V(G^i)$; or $A^i_1 \cap A^i_2 = \emptyset$ and there exists j_2 such that $A^i_2 \cap N_{j_2}(A^i_1) \neq \emptyset$, $A^i_2 \subseteq N_{j_2-1}(A^i_1) \cup N_{j_2}(A^i_1)$ and $N_{j_2+1}(A^i_1) = \emptyset$, where $N_j(A^i_1)$ is the j-th neighborhood of A^i_1 in G^i (and, analogously, there exists j_1 such that $A^i_1 \cap N_{j_1}(A^i_2) \neq \emptyset$, $A^i_1 \subseteq N_{j_1-1}(A^i_2) \cup N_{j_1}(A^i_2)$ and $N_{j_1+1}(A^i_2) = \emptyset$). Besides, each vertex in A has a neighbor in $V(G^i) \setminus A$, for each clique $A \in \mathcal{A}^i$. Finally, if A^i_1 and A^i_2 are in the same set of $\mathcal{P}$, then A^i_1 is anticomplete to A^i_2.*

Now suppose that we are given a strip decomposition obeying to Theorem 3 for a claw-free *perfect* graph G. If we are interested in finding a MWCC of G, following Theorem 2, we must show that for a strip that is distance simplicial we can compute in polynomial time: a MWCC of the strip; a MWCC of $G^i_\bullet$, i.e. G^i plus a vertex complete to both extremities, when the strip $(G^i, \mathcal{A}^i)$ is odd-short; a MWCC of $G^i_=$, i.e. G^i plus the edges joining the extremities A^i_1, A^i_2 of the strip, when they are in the same class of the partition and there is an A_1–A_2 path of length two.

We start by briefly describing how to compute a MWCC in distance simplicial graphs (recall that they are indeed perfect). We rely on a property of perfect graphs, namely, there always exists a clique which intersects each MWSS: we will call such a clique *crucial* (crucial cliques are a key ingredient to the algorithm in [10]). Our algorithm relies on the fact that for graphs that are distance simplicial w.r.t. some identifiable clique K, we can inductively compute crucial cliques and decide the value of this clique in a MWCC. The first crucial clique will be $K' := K \cup \{v \notin K : v \text{ is complete to } K\}$: we will suitably update the weight of each vertex, and then find a new crucial clique (w.r.t. the new weights) in an inductive way.

We now show that, for an odd-short distance simplicial strip H_i, we can compute in polynomial time a MWCC of $G^i_\bullet$. Note that, in this case $G^i_\bullet$ is claw-free and, following Theorem 2, perfect. In this case, we prove that $G^i_\bullet$ is cobipartite.

Note that, if $G^i_\bullet$ is cobipartite, then it is distance simplicial w.r.t. each of the two cliques covering its vertex set, so a MWCC can be found as above. We now show that, for a distance simplicial strip H_i, such that the extremities are in the same class of the partition and there is an A_1–A_2 path of length two, we can compute in polynomial time a MWCC for $G^i_=$. Note that, in this case, $G^i_=$ is claw-free and, following Theorem 2, we may assume that it is perfect and that $\alpha_{w^i}(G^i_=) = \alpha_{w^i}(G^i \setminus (A^i_1 \cup A^i_2))$ holds, where w^i is the weight function defined on the vertices of G^i (that w.l.o.g. we take strictly positive, i.e., we remove vertices with $w^i(v) = 0$). In this case, we prove that either $G^i_=$ is cobipartite, or every MWCC of G^i is also a MWCC of $G^i_=$. If $G^i_=$ is not cobipartite, then we may simply ignore the edges between the two extremities of the strip and then compute a MWCC in G^i, which is distance simplicial.

We have therefore the following theorem for strip-composed claw-free perfect graphs. We underline that the resulting algorithm never requires the computation of any MWSS on the strips, while it uses a primal-dual algorithm for the maximum weighted matching on the root graph of $\tilde{G}$ (see Section 3).

Theorem 4. *Let G be a claw-free perfect graph with a non-negative weight function w on $V(G)$ and let G be as in case (ii) of Theorem 3. Then we can compute a* MWCC *of G w.r.t. w in time $O(|V(G)|^3)$.*

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