Computational complexity of edge modification problems in different classes of graphs

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Abstract

Edge modification problems in graphs have a lot of applications in different areas. Polynomial time algorithms and NP-completeness results appear in several works in the literature. In this paper, we prove new complexity results of these problems in some graph classes, such as, interval, circular-arc, permutation, circle, bridge and weakly chordal graphs.

Key words: computational complexity, edge modification problems, graph classes.

1 Introduction

Edge modification problems include completion, deletion and editing problems. Let $G = (V, E)$ be a graph, a graph property π (for example if the graph

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belongs to a certain class) and an integer number k . In the π-editing decision problem we need to know if there exists a set $F \subseteq V \times V$ with $|F| \leq k$, such that the graph $G' = (V, E \triangle F)$ satisfies π ($E \triangle F$ denotes the symmetric difference between E and F). The π-deletion decision problem only allows edge deletions ($F \subseteq E$) and the π-completion decision problem only permits to add edges ($F \cap E = \emptyset$).

Edge modification problems have applications in several areas, such as, molecular biology and numerical algebra (see, for example, [1,11,13,19]).

Physical Mapping is a central problem in molecular biology and the human genome project. The problem is to reconstruct the relative position of fragments of DNA along the genome from information on their pairwise overlaps. A simplified model of the problem which consider false negative and positive errors in the experimental data, leads to edge-modification problems in interval or unit interval graphs [11].

The bacterial DNA and cytoplasmic DNA in animals exist in a closed circular form. Furthermore, giant DNA molecules in higher organisms form loop structures held together by protein fasteners in which each loop is largely analogous to closed circular DNA. In this case, similar models lead to edge-modification problems in circular-arc graphs.

Computational complexity of editing, deletion and completion problems in graph classes has been widely studied in the literature [6,9,11,14,16,18,20,24,25].

In this work, we prove new complexity results of these problems in some classes of graphs, such as, interval, circular-arc, permutation, circle, bridge and weakly chordal graphs.

Table 1 summarizes known results about edge modification problems, including those obtained in this work (boldfaced). Open problems are indicated by "?".

Graph classes and graph theory properties not defined here can be found in [3], [8] or [12].

2 Main results

First three theorems show polynomial reductions from hamiltonian path restricted to cubic planar graphs which is NP-complete [10], to interval containment deletion, interval containment editing and interval editing, respectively.

Given a cubic graph G (all its vertices with degree 3), we define the triangle

Graph Classes	**Completion**	**Deletion**	**Editing**
Perfect	NPC [18]	NPC [18]	NPC [18]
Chordal	NPC [24]	NPC [20]	NPC [20]
Interval	NPC [8,24]	NPC [11]	**NPC**
Unit Interval	NPC [24]	NPC [11]	**NPC**
Circular-Arc	NPC [20]	NPC [20]	**NPC**
Unit Circular-Arc	NPC [20]	NPC [20]	**NPC**
Proper Circular-Arc	NPC [20]	NPC [20]	**NPC**
Chain	NPC [24]	NPC [20]	?
Comparability	NPC [14]	NPC [25]	NPC [18]
Cograph	NPC [6]	NPC [6]	?
AT-Free	?	NPC [20]	?
Threshold	NPC [16]	NPC [16]	?
Bipartite	irrelevant	NPC [9]	NPC [9]
Split	NPC [18]	NPC [18]	P [15]
Cluster	P [20]	NPC [6]	NPC [20]
Trivially Perfect	NPC [24]	NPC [20]	?
Permutation	**NPC**	**NPC**	**NPC**
Circle	**NPC**	**NPC**	**NPC**
Weakly Chordal	**NPC**	**NPC**	?
Bridge	?	**NPC**	?

Table 1
Summary of complexity results

graph $\widehat{G}$ as a new cubic graph obtained from G replacing each vertex by a triangle and using the edges (v_i, v_j) of G to join a representant of v_i with a representant of v_j in $\widehat{G}$. It is easy to see that if G is a planar graph, so it is $\widehat{G}$.

Theorem 1 *Let G be a cubic planar graph, G has a hamiltonian path if and only if its triangle graph $\widehat{G}$ has an interval containment subgraph with at least $4n-1$ edges.*

Theorem 2 *Let $G = (V, E)$ be a cubic planar graph and $\widehat{G} = (\widehat{V}, \widehat{E})$ its triangle graph, G has a hamiltonian path if and only if there is an edge set F such that $\widetilde{G} = (\widehat{V}, \widehat{E} \triangle F)$ is an interval containment graph and $|F| \leq \frac{n}{2} + 1$.*

Theorem 3 *Let $G = (V, E)$ be a cubic planar graph and $\widehat{G} = (\widehat{V}, \widehat{E})$ its triangle graph, G has a hamiltonian path if and only if there is an edge set F such that $\widetilde{G} = (\widehat{V}, \widehat{E} \triangle F)$ is an interval graph and $|F| \leq \frac{n}{2} + 1$.*

Using the equivalence between permutation graphs and interval containment graphs [5], and the existence of polynomial time recognition algorithms for permutation graphs [17] and interval graphs [2], we have the following corollary.

Corollary 4 *Permutation deletion, permutation editing and interval editing are NP-complete problems.*

As a collateral result of the proof of theorem 3, this corollary can be proved.

Corollary 5 *Unit interval editing is NP-complete.*

Next theorem can be proved using the NP-completeness of chain deletion [18], and the existence of polynomial time recognition algorithms for bridge graphs [7] and weakly chordal graphs [22].

Theorem 6 *Bridge deletion and weakly chordal deletion are NP-complete problems.*

Since permutation graphs and weakly chordal graphs are closed under complement [3], we have the following corollary.

Corollary 7 *Permutation completion and weakly chordal completion are NP-complete.*

This proposition is showed in [18].

Proposition 8 *If π and π' are graph properties such that for every graph G and a stable set S, G satisfies π if and only if $G \cup S$ (the disjoint union between graphs G and S) satisfies π', then π-deletion is polynomially reducible to π'-deletion. If in addition π is hereditary, then π-completion (π-editing) is polynomially reducible to π'-completion (π'-editing).*

A graph G is a (unit)interval graph if and only if $G \cup S$ is a (unit)circular-arc graph, for any stable set S. Additionally, (unit)interval is a hereditary class of graphs. So, by Proposition 8, (unit)interval edge modification problems are polynomially reducible to the corresponding (unit)circular-arc edge modification problems. Since (unit)circular-arc graphs can be recognized polynomially [4], we have the following result.

Corollary 9 *Circular-arc and unit circular-arc editing are NP-complete.*

Since proper interval graphs are equivalent to unit interval graphs [3] and proper circular-arc graphs have polynomial time recognition [23], using proposition 8, we have the following result.

Corollary 10 *Proper circular-arc editing is NP-complete.*

A trivial corollary of Proposition 8 can be proved applying that result to the complement of the graph.

Proposition 11 *If π and π' are graph properties such that for every graph G and a complete K, G satisfies π if and only if $G + K$ (the union between graphs G and K, joining every vertex of G with every vertex of K) satisfies π', then π-deletion is polynomially reducible to π'-deletion. If in addition π is hereditary, then π-completion (π-editing) is polynomially reducible to π'-completion (π'-editing).*

A graph G is a permutation graph if and only if $G + K$ is a circle graph, for any complete K [12]. Additionally, permutation graphs are a hereditary class of graphs. So, permutation edge modification problems are polynomially reducible to the corresponding circle edge modification problems. Since circle graphs can be recognized polynomially [21], we have the following result as a consequence of Proposition 11.

Corollary 12 *Circle completion, deletion and editing are NP-complete problems.*

Note 1 *Reductions involved in the proofs of theorems 1, 2 and 3 imply that the NP-completeness results obtained in this work hold even when the input graphs are restricted to be cubic planar graphs.*

References

[1] H. Bodlaender and B. De Fluiter, On intervalizing k-colored graphs for DNA physical mapping, *Disc. App. Math.* **71** (1966) 55–77.

[2] K. Booth and G. Lueker, Testing for the consecutive ones property, interval graphs, and graph planarity using PQ-tree algorithms, *J. Comp. Sci.* **13** (1976) 335–379.

[3] A. Brandstädt, V. Le and J. Spinrad, *Graph Classes: A Survey* (SIAM, 1999).

[4] G. Durán, A. Gravano, J. Spinrad and A. Tucker, *Polynomial time recognition of unit-circular arc graphs*, submitted.

[5] B. Dushnik and E. Miller, Partially ordered sets, *Amer. J. Math.* **63** (1941) 600–610.

[6] E. El-Mallah and C. Colbourn, The complexity of some edge deletion problems, *IEEE Trans. Circ. Syst.* **35** (3) (1988) 354–362.

[7] M. Farber and R. Jamison, On local convexity in graphs, *Disc. Math.* **66** (1987) 231–247.

[8] M. Garey and D. Jonhson, *Computers and Intractability: A Guide to the Theory of NP-Completeness* (Freeman and Co., 1979).

[9] M. Garey, D. Johnson and L. Stockmeyer, Some simplified NP-complete graph problems, *Theor. Comp. Sci.* **1** (1976) 237–267.

[10] M. Garey, D. Johnson and R. Tarjan, The planar Hamiltonian circuit problem is NP-complete, *SIAM J. Comp.* **5** (1976) 704–714.

[11] P. Goldberg, M. Golumbic, H. Kaplan and R. Shamir, Four strikes against physical mapping of DNA, *J. Comp. Biol.* **2** (1995) 139–152.

[12] M. Golumbic, *Algorithmic Graph Theory and Perfect Graphs* (Acad. Press, 1980).

[13] M. Golumbic, H. Kaplan and R. Shamir, On the complexity of DNA physical mapping, *Advances in App. Math.* **15** (1994) 251–261.

[14] S. Hakimi, E. Schmeichel and N. Young, Orienting graphs to optimize reachability, *Inform. Proc. Letters* **63** (1997) 229–235.

[15] P. Hammer and B. Simeone, The splittance of a graph, *Combinatorica* **1** (1981) 275–284.

[16] F. Margot, Some complexity results about threshold graphs, *Disc. App. Math.* **49** (1994) 229–308.

[17] R. McConnell and J. Spinrad, Linear-time transitive orientation, in: *Proc. 8th Ann. ACM-SIAM Symp. on Disc. Algorithms* (New Orleans, 1997), 19–25.

[18] A. Natanzon, R. Shamir and R. Sharan, Complexity classification of some edge modifications problems, *Disc. App. Math.* **113** (2001) 109–128.

[19] J. Rose, A graph-theoretic study of the numerical solution of sparse positive definite systems of linear equations, in: R.C. Reed, ed., *Graph Theory and Computing* (Acad. Press, 1972) 183–217.

[20] R. Sharan, *Graph Modification Problems and their Applications to Genomic Research*, Phd. Thesis, Tel-Aviv University (2002).

[21] J. Spinrad, Recognition of circle graphs, *J. of Algorithms* **16** (2) (1994) 264–282.

[22] J. Spinrad and R. Sritharan, Algorithms for weakly triangulated graphs, *Disc. App. Math.* **59** (1995) 181–191.

[23] A. Tucker, Matrix characterizations of circular-arc graphs, *Pacific J. Math.* **38** (1971) 535–545.

[24] M. Yannakakis, Computing the minimum fill-in is NP-complete, *SIAM J. Alg. Disc. Math.* **2** (1) (1981) 77–79.

[25] M. Yannakakis, Edge deletion problems, *SIAM J. Comp.* **10** (2) (1981) 297–309.