An application of the traveling tournament problem: The Argentine volleyball league

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Abstract

This article describes the optimization process used to schedule the First Division of Argentina's professional volleyball league. The teams in the league are grouped into couples and matches are held on Thursdays and Saturdays. In every pair of consecutive Thursday-Saturday matches, the two teams in each couple play against two teams from another couple. Minimization of travel distances is critical since the teams' home locations are scattered throughout the country and teams do not return their home sites between consecutive away matches, making this problem a variation of the well-known traveling tournament problem. The coupled format gives rise to two key decisions: (a) how to couple the teams and (b) how to schedule the matches. We apply integer programming techniques and a tabu search heuristic to solve these issues. The resulting schedules have been successfully used in the 2007-2008, 2008-2009, 2009-2010, and 2010-2011 league

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seasons, reducing the total travel distance while meeting all of the teams' requirements. This is the first reported application of the traveling tournament problem to a real-world sports league in the optimization literature.

Key words: sports scheduling, integer programming, traveling tournament problem, team couples, volleyball

1 Introduction

In recent years, *sports scheduling* has become a very active field in the operations research community and a source of problems that are both interesting and challenging. In practice, the associated combinatorial optimization problems are usually very difficult to solve, albeit for reasons not yet fully understood. Moreover, for many sports leagues a central objective is to minimize costs or travel distances, and this generates problems whose solution is much harder still.

Applications of operations research techniques have been reported for many real-world sports leagues such as soccer [1, 13, 19, 31], basketball [27, 42], hockey [17, 29] and cricket [41]. Since each league has its particular characteristics, different models, algorithms and methodological tools have been studied and proposed in the literature (see, e.g., [3, 5, 8, 9, 10, 11, 14, 18, 22, 28, 34, 36]). Informative surveys of sports scheduling can be found in [16, 24, 32] and benchmark instances from some applications are given in [30].

A central issue in sports scheduling is the well-known *traveling tournament problem* (TTP) [14]. Given a set of n teams and the travel distances between every pair of teams, the problem consists in scheduling a *double round-robin* tournament (i.e., each team plays against every other team exactly twice, once at home and once away) with $2(n-1)$ time slots such that no team plays fewer than L nor more than U home (resp. away) matches in a row (typically, $L = 1$ and $U = 3$), no two teams play against each other in two consecutive time slots, and the total travel distance is minimized. It is further assumed that no team returns home between consecutive away matches. The TTP is an extremely hard combinatorial optimization problem and there are open instances with only $n = 12$ teams [38]. Interestingly, understanding the computational complexity of the TTP turns out to be a difficult task. The

TTP is known to be NP-hard if $L = 1$ and $U = 3$ [37] and if $L = 1$ and $U = \infty$ [2]. The complexity for other values of L and U is still open, however [39].

There have been a number of recent computational developments aimed at tackling the TTP in practice. In [7] the author proposes a Benders approach that allows strong lower bounds to be computed for benchmark instances. In [15, 23] an extensive formulation is suggested that contains one variable for every path in a time-discrete network representing each team's road trips, and is solved by branch-and-price procedures. An application of the DFS* search procedure to the TTP, presented in [40], exactly solves previously unsolved instances with very good execution times. Instances of up to 6 teams can be successfully handled using quite standard algorithmic machinery [38].

In this work we describe the model-based scheduling of the regular phase of the Argentine men's volleyball league First Division using integer programming and tabu search techniques. League play is organized according to the *coupled format* in which the teams are divided into couples that are geographically close and the matches are grouped into pairs of temporally close meetings. This arrangement has been previously addressed in the combinatorial optimization literature only in [18], where some variations on this setup used by the Czech national basketball league are examined.

Since the teams in the Argentine volleyball league are scattered throughout the country and road trips are usually made by bus, the main objective of the league's scheduling process is to adequately manage the travel distances. Thus, the scheduling problem is a practical application of the TTP. The schedules obtained by the integer programming and tabu search techniques described in this work were successfully applied in the 2007-2008, 2008-2009, 2009-2010, and 2010-2011 seasons and schedules based on similar techniques were employed for the men's Second Division and women's First Division of the league. This is the first real-world application of the TTP reported in the literature [39].

The remainder of this paper is organized as follows. Section *The league* describes in detail the league format and the particular characteristics arising from the use of couples of teams and pairs of matches. Section *The scheduling process* describes the planning process and Section *Results* reports on the schedules obtained. The paper closes with some concluding remarks followed by two appendices setting out the complete integer programming models and the tabu search techniques employed during the volleyball league scheduling process.

2 The league

The First Division of the Argentine volleyball league consisted of 12 teams until 2008-2009 when the number dropped to 11. For the 2010-2011 season, the league returned to 12 clubs. The league season consists of a *regular phase* followed by *playoffs*. The regular phase is a double round-robin tournament, the top eight teams qualifying for the first playoff series which is a best-of-five quarter-finals. The winners proceed to the best-of-five semi-finals and then a best-of-seven final to determine the champion.

A distinctive characteristic of the league is the *coupling of teams*. Under this system, teams are grouped into geographically close pairs. In 2007-2008 and 2010-2011, when the league had 12 teams, there were six such *couples* (see Figure 1 for the 2007-2008 couples). In 2008-2009 and 2009-2010 the 11 clubs were divided into five couples and one *uncoupled team*. The team in a couple that placed higher in the regular phase of the previous season is called the *A-team*, the other then being referred to as the *B-team*.

Matches are usually held on Thursdays and Saturdays and are also grouped into pairs, each Thursday game and the one played on the immediately following Saturday forming a *weekend*. Every weekend, half of the couples visit a couple from the other half, each visiting couple team playing each of the two home couple teams that are hosting them. On the Thursday, the A-team from each couple plays against the B-team from the other couple, and on the Saturday the two B-teams and the two A-teams play each other. This prespecified setup for the matches between the visiting and home couples on a given weekend is called the *visiting schema*. In the case where $n = 11$, on a weekend when a couple plays against the uncoupled team, each couple member plays exactly one match.

There are two special weekends, called *intra-couple weekends*, on which the two teams in each couple play against each other. The coupled teams play only once on these weekends and the five or six possible matches across the league (depending on whether there are 11 or 12 teams) are uniformly distributed between Thursday and Saturday. The uncoupled team,

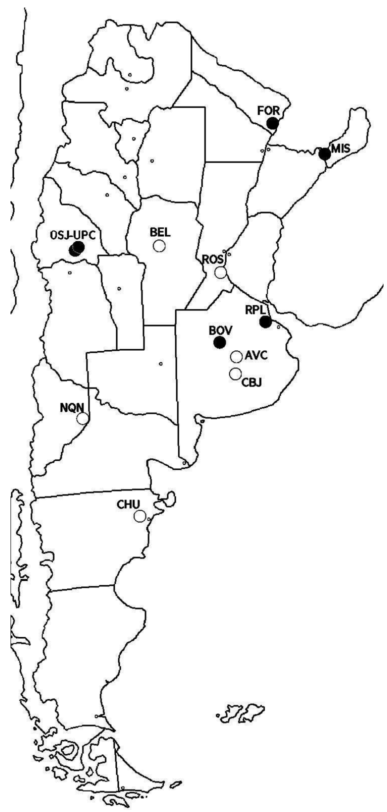


Figure 1: Teams and team couples for the 2007-2008 season. The couples are FOR-MIS, BEL-ROS, OSJ-UPC (located in the same city), BOV-RPL, CBJ-AVC and NQN-CHU. The average distance between teams of the same couple is 342 kilometers while the longest intra-couple distance (NQN-CHU) is 700 kilometers. The longest distance between any two teams is the 2,365 kilometers separating FOR and CHU.

meanwhile, has a bye. The regular phase of the season for each team will thus comprise either 20 matches (in the case of 11 teams) or 22 matches (in the case of 12 teams) played over a span of 12 weekends.

The season format as just described is a variation on a double round-robin tournament that we propose to call a *coupled double round-robin*. In 2007-2008 and 2008-2009, the intra-couple weekends were played on weekends 1 and 7 and the schedule was mirrored (i.e., the schedule for the second half of the regular phase was the same as for the first half but with the teams' home-away status reversed). In 2009-2010, the intra-couple weekends were held on weekends 6 and 12 and a non-mirrored schedule was used. In 2010-2011, the schedule was again mirrored and the intra-couple weekends were held on weekends 1 and 12.

A drawback with the format when there are an odd number of teams is that at any given time the different teams will generally not have played the same number of matches so that the standings may not properly reflect each team's performance up to that point. This is so because the standings are based on earned points rather than winning percentage. Thus, a team may have won all of its matches yet will not necessarily be in first place.

In addition to the Argentine volleyball league, this coupled format is employed by Argentina's first and second division basketball leagues and, with some minor modifications, the highly ranked Brazilian professional volleyball league and the Czech national basketball league [18]. Many American college sports leagues also use a coupled arrangement, the best-known example being the Pacific-10 (PAC-10) college athletic conference. Three properties of this format type are particularly interesting from a scheduling point of view:

- *It reduces the manual/computational burden of generating a schedule.* If we disregard the intra-couple weekends, the task of generating a feasible coupled double round-robin schedule for n teams is equivalent to generating a double round-robin schedule for $n/2$ teams (each couple treated as a single "team"). This feature is not only valuable for defining schedules manually, but is also very helpful in that it allows solutions to be found for many problems that would otherwise be computationally intractable such as the design of a schedule minimizing total travel distance (recall that TTP instances with 10 or more teams may be very hard to solve to optimality).

- *It introduces a simple but effective fair competition criterion.* In the Argentine volleyball league there are usually one to three very strong teams (usually *Drean Bolívar*, *La Unión de Formosa* and *UPCN Vóley*, identified in Figure 1 as BOV, FOR and UPC, respectively). If no couple has more than one strong team (which has been the case in recent years), the coupled format ensures that no team will have to play against two strong teams on the same weekend. This is a simple but very easily explained fair competition feature. Note, however, that if two strong teams do form a couple, the format for the remaining teams will be unbalanced. Such couplings should therefore be avoided.

- *It contributes to good management of travel distances.* Although the coupled format imposes an additional constraint on the league's standard TTP format, assigning geographically close teams to each couple can help keep travel distances under control. In our experience, the combination of reasonably designed couples and a carefully crafted schedule generates acceptable travel distances. For the instances considered in this work, the best (not necessarily optimal) solutions found by the tabu search heuristic described in Appendix 2 for the standard uncoupled TTP with 12 teams are no more than 1% better than the solutions that we found for the coupled TTP with 6 team couples. This provides empirical evidence that the travel distances are not greatly affected by the coupling of teams.

In the last few years, interest in the national volleyball league among the general public has increased significantly thanks to growing media coverage and the recent revitalization of Argentina's national teams. The junior and youth teams won bronze medals in their respective 2009 World Cups while the senior team achieved an all-time record 5th place finish in the 2009 World League and came in 9th at the 2010 World Cup, returning to the top ten after an absence of 8 years. As a result, a number of provincial governments and private sponsors have put up funding for competitive teams, which in turn have requested a transparent and well-designed schedule.

The main objective of the schedule design is to minimize the total travel distance. For the 2009-2010 and 2010-2011 seasons, travel equity considerations were also taken into account.

As already noted, travel distances are a constraining issue for the Argentine volleyball league due to the scattered locations of the teams around the country. The maximum distance between any two teams is that separating FOR and CHU (see Figure 1), which are 2,365 kilometers apart (about the distance from New York City to Dallas or Madrid to Hamburg). The teams usually travel by bus, and with a few exceptions do not return home between two consecutive away weekends. This condition also holds in the standard setting of the TTP [14].

In addition, no team can play more than two consecutive home or two consecutive away weekends (not counting intra-couple weekends) and no two couples may play each other twice on consecutive weekends (in the case of a mirrored schedule this constraint is trivially satisfied). Disregarding the intra-couple weekend, this problem is a special case of the TTP, with couples instead of teams and pairs of matches (i.e., weekends) instead of single matches. Under this arrangement, $L = 1$ and $U = 2$ (at most, two consecutive home and away weekends). Interestingly, the $U = 2$ condition appeared previously in [35].

3 The scheduling process

The Argentine volleyball league is managed by the *Asociación de Clubes Liga Argentina de Vóleibol* (ACLAV), a nonprofit organization owned by the league teams themselves. The Association employs a *competition manager* who is responsible for all organizational aspects of the league including scheduling design. The competition manager reports to the ACLAV Council which is composed of one representative from each team. The competition manager is also ACLAV's contact person for the authors of the present study, who are part of a team of researchers based at the University of Buenos Aires and the National University of General Sarmiento.

The scheduling of the league is built around two key decisions: (a) how to couple the teams, and (b) how to schedule the matches between the team couples. One way of approaching these decisions is to devise an integer programming model that addresses both of them at the same time. However, a straightforward formulation that simultaneously attempts to (a) design the team couples and (b) schedule the matches so as to minimize the total travel distance performs very poorly with *Cplex* [21]; indeed, the software quite often fails to find feasible solutions even after several hours. Although the TTP for $U = 2$ is not known to be NP-hard, we conjecture that this combined problem is computationally hard.

In light of the above observations, we did not explore the simultaneous approach further, concentrating instead on an intuitive two-stage process. In Stage 1 we design the team couples while in Stage 2 we schedule the matches for the couples specified in Stage 1. As we report in the present section and Appendix 1, this method turned out to be computationally feasible in the sense that each stage can be solved either to optimality (for the couple design problem, the meaning of "optimality" must be precisely defined since finding a proper objective function is a non-trivial task) or with near-to-optimal results. Furthermore, it has the advantage that the computational procedure involved is readily explained to the team representatives, a crucial issue in the interaction between the schedulers and the teams given the importance of ensuring the overall process is transparent.

3.1 Stage 1: Designing the team couples

The first stage in the scheduling process must determine a *coupling* of the teams (a perfect matching in the case of 12-team league, and a matching of 10 teams in the case of an 11-team league) such that the second stage can generate a schedule with a minimum or near-minimum total travel distance. The usual constraints for the first stage involve pairs of teams that should not be coupled, typically to avoid long Thursday-Saturday trips for the visiting teams, to keep strong teams in different couples, or incorporate certain public attendance considerations. The latter would apply, for example, to pairs of teams whose home matches attract many of the same spectators, as coupling them would force fans to choose just one of the games whenever the pair played at home.

The simplest approach to team coupling is to determine the minimum-weight matching on the complete graph whose vertices represent the teams and whose edge weights represent the travel distances between them [26]. However, this approach only takes into account the travel distances between teams in the same couple, failing to consider the travel distances between different couples.

The best team coupling is clearly one that generates a minimum-distance schedule. However, trying to find such an arrangement just amounts to attempting the combined approach described above, which unfortunately could not be successfully carried through in practice. Since the 2009-2010 season we have therefore resorted to a procedure that includes travel distance considerations without guaranteeing that the resulting coupling yields a minimum-distance schedule. In the previous editions, the coupling was perfomed manually by the competition manager.

Let (A, B) be a team couple and M be the set of team couples for a league. Throughout the season, couple (A, B) will make a number of *tours* on which it plays away matches against the other couples. Each tour is composed of at most two consecutive weekends, after which couple (A, B) must return home. Let $\gamma_M(A, B)$ be the optimal travel distance for couple (A, B) in M, i.e., the minimum possible sum of the travel distances to and from the away matches played by A and B in any feasible schedule. If the distance matrix satisfies the triangle inequality and there are 12 teams, then any two single-weekend tours can be combined into one two-weekend tour without increasing the total travel distance. Therefore, the optimal travel distance $\gamma_M(A, B)$ for couple (A, B) can be obtained from a set of tours composed of two four-match tours and one three-match tour, the latter comprising one away weekend and the away intra-couple match.

We propose to search for a coupling M that minimizes $\gamma_M := \sum_{(A,B)\in M} \gamma_M(A, B)$. In other words, we set out to find a coupling M such that the total travel distance of the optimal tours for each couple in M is at a minimum. We call this problem the *optimal-tour coupling problem*. Note that the optimal sets of tours (i.e., tours with distance $\gamma_M(A, B)$ for each couple (A, B) in M) in general cannot be combined into a feasible schedule, as in any feasible schedule, at least one couple must perform two single-weekend tours and the optimal tours are composed by three two-weekend tours. Hence, the value $\min_M \gamma_M$ provides a lower bound to the total travel distance in any feasible schedule. The model presented in Appendix 1 provides an integer programming formulation for the optimal-tour coupling problem which, by minimizing γ_M over all possible couplings M, tries to obtain a good coupling for Stage 2.

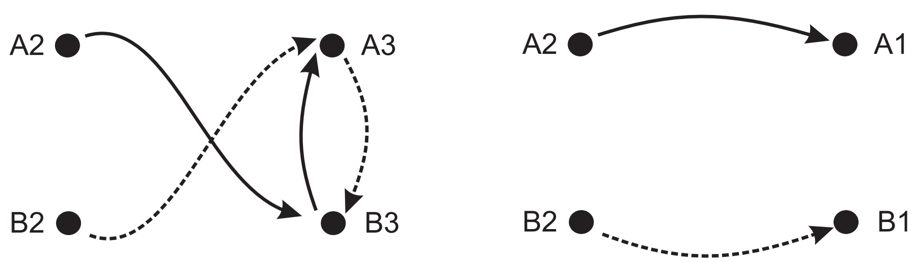
Note that this problem is not equivalent to finding a perfect matching on a precomputed graph, as the distance $\gamma_M(A, B)$ traveled by a couple (A, B) depends on the complete coupling/matching M. For some instances the optimal solution will nevertheless coincide with the optimal perfect matching in the graph representing the teams. Appendix 1 presents a straightforward integer programming model for the optimal-tour coupling problem that produces computational results which, though reasonable, are not optimal. However, by adding symmetry-breaking constraints [33] and families of valid *equations* to the initial formulation we were able to find optimal solutions for 11 and 12 teams. An optimal solution to the problem provides a coupling that may not generate the optimal coupled schedule in terms of total travel distance but will take into account inter-couple trips, which are not considered in a simple minimum-weight matching.

In addition to the coupling generated by this integer programming approach, the competition manager and the team representatives usually propose alternative couplings that are slight variations on the model-generated solution.

3.2 Stage 2: Scheduling the matches

For each team coupling considered, the procedures in Stage 2 attempt to generate a schedule. Since the team couples are given and on each weekend each couple plays some other couple, we can model this problem as a TTP with six "teams" (each corresponding to a couple), with the additional feature that on certain prespecified dates (the intra-couple weekends) each "team" must play at home. Note that the prespecified visiting schema allows us to make this reduction to six "teams" without loss of generality.

More specifically, when the "team" representing couple $(A1, B1)$ travels from $(A2, B2)$ to $(A3, B3)$, team $A1$ travels from $A2$ to $B3$ for the Thursday match and from $B3$ to $A3$ for the Saturday match while the team $B1$ travels from $B2$ to $A3$ for the Thursday match and from $A3$ to $B3$ for the Saturday match (see Figure 2a). The total distance traveled by couple $(A1, B1)$ to play an away match against $(A3, B3)$ is thus $2d_{A3,B3} + d_{A2,B3} + d_{B2,A3}$. If the couple $(A1, B1)$ plays a weekend at home and then travels to $(A3, B3)$ then the total distance admits a similar formula, i.e., the total distance is $2d_{A3,B3} + d_{A1,B3} + d_{B1,A3}$. On the other hand, if $(A1, B1)$ performs a trip back home after an away weekend at $(A2, B2)$, then the total distance equals $d_{A2,A1} + d_{B2,B1}$ (see Figure 2b). Thus, the distance matrix between the



(a) (b)

Figure 2: Trips for couple $(A1, B1)$ when, after playing $(A2, B2)$, (a) it plays $(A3, B3)$ or (b) it returns home. The solid lines represent trips by team $A1$ and the dotted lines represent trips by team $B1$.

six "teams" representing the couples is well-defined and the reduction to a six-team TTP can be carried out effectively.

The home-away status of the teams on the intra-couple weekends is not modeled explicitly. Rather, we specify beforehand that on the first such weekend the B-team of each couple plays at home against the A-team and on the second one the opposite is the case. In practice, the competition manager may change the home-away status of some of these matches in response to special considerations without significantly affecting the total travel distance.

Various other constraints are typically included in the actual league scheduling, a reflection of the fluctuating conditions that generally prevail in the days immediately before the announcement of the definitive schedule. The ability of the method to quickly generate new schedules incorporating changes in the constraints proved to be a major benefit for the competition manager. Selecting a suitable set of constraints involves experimenting with 10 to 20 models with different sets. Since some of the constraints will usually turn out to be mutually exclusive, arriving at a set that meets the competition manager's requirements while also generating a feasible model is a process of trial-and-error. Examples of these requirements include the following:

- *Each couple must play one of weekends 2 and 3 at home and the other away.* This condition was applied in the 2007-2008 mirrored-format season. Since weekends 1 and 7 were the intra-couple weekends and there was a two-week holiday period before weekend

7, this additional constraint ensured that no team played three consecutive weekends away –including the intra-couple weekend– after the holiday period.

- *Certain teams cannot use their stadium on prespecified weekends.* Although this situation did not arise in 2009-2010, in the other seasons several teams were unable to play at home on certain weekends. This usually occurred because some other local sports team (e.g., basketball) had booked the stadium on those dates. In our experience with the ACLAV instances, solution times tend to be much shorter when there are many such constraints. This is a major advantage over manual scheduling procedures for which the presence of this type of constraint can be a tremendous burden on the process of generating a feasible result.

- *The matches on a prespecified weekend must be played near a certain city.* Since 2008-2009 a special short tournament called the "Super 8" is held during the season, interrupting the regular schedule for a week. Since this tournament takes place in one or two cities and starts on a Tuesday, the scheduling process attempted to avoid setting matches for the previous weekend located far from the Super 8 venues. The constraint for this requirement can be written as a special case of the previous one.

The schedules for 2007-2008, 2008-2009, and 2010-2011 were mirrored formats with the constraint that no team would play more than two consecutive away weekends. The restriction did not apply to the intra-couple weekends, which were played on weekends 1 and 7. Since in any intra-couple match one team plays away, some teams have to play at home or away on up to three consecutive weekends including the intra-couple weekend 7. Unfortunately, this situation is mathematically unavoidable for a coupled double round-robin mirrored schedule with six team couples. Indeed, suppose that there exists such a mirrored schedule in which no team plays three home or away consecutive weekends. The three couples playing at home on weekend 2 must play away (resp. home) on weekend 5 (resp. 6) in order to avoid one of the teams from each couple having to play at home (resp. away) on weekends 5, 6, and 7 (resp. 6, 7, and 8). Since these three couples have the same home-away status in the weekends 2, 5, and 6, that leaves only weekends 3 and 4 for them to play each other in the first round, a contradiction as we need at least three weekends for three couples to confront each other. As a result, a non-mirrored schedule was designed for the 2009-2010 league and the condition imposing no more than two consecutive away weekends was extended to include the intra-couple weekends.

Since both mirrored and non-mirrored schedules may be requested, we have implemented both an integer programming approach and a tabu search metaheuristic for the match scheduling problem. It turns out that the mirrored version is solvable to optimality with *Cplex* [21] in anywhere from one minute to six hours (depending on the number of home-away constraints), but the inherent complexity of the non-mirrored case forces us to resort to heuristic techniques. Our tabu search heuristic is adapted from the one used for the TTP posed in [6], which was able to find optimal solutions for the non-mirrored six-team instances in [38] and is therefore a reasonable choice for solving the non-mirrored case. In view of this we opted not to turn to more sophisticated solution techniques such as those given in [4, 40]. Moreover, the approaches we decided upon readily permit constraints such as those described above to be added or removed, a crucial feature in a practical scheduling environment.

Several objective functions have been used in the scheduling of the various volleyball league seasons:

- *Minimize the total travel distance.* This was applied for the 2007-2008 and 2008-2009 schedules and is the usual objective in TTP instances. As a minor variation, in 2008-2009 distances in excess of 1,080 kilometers on all point-to-point travel were counted double in order to penalize bus trips longer than 12 hours (the minimum time needed to travel 1,080 kilometers given the legal limit of 90 kilometers per hour for intercity buses in Argentina).

- *Minimize the distance of the* most traveled *team.* This objective, easily implemented in an integer programming model, attempts to evenly distribute the travel distances of the teams at the expense of total travel distance. The resulting schedules are usually not acceptable since the solution only involves the travel of the outlying teams while the travel patterns of the more centrally located teams are likely to be inefficient. Moreover, the total travel distances tend to be larger. For these reasons, this objective function was not further explored.

- *Minimize the distance gap between the* most traveled *team and the* least traveled *team.* Again, this objective function seeks to evenly distribute the travel distances, but usually generates schedules with longer distances for the centrally located teams (to bring their total up to the level of the outlying teams). Furthermore, the running times to optimality tend to be much greater. On these grounds, this objective function was dropped during preliminary experimentation.

- *Minimize a combination of total travel distance and the distance of the most traveled team.* If the total travel distance is z_{TT}, the distance of the most traveled team is z_{MT} and there are n teams, then the objective function is defined as $z = z_{TT} + n\, z_{MT}$. This specification tries to optimize the total travel distances while maintaining a certain degree of equity among the teams. It was used for the construction of the 2009-2010 and 2010-2011 schedules. We are not aware of previous works utilizing this formulation for the TTP.

Our integer programming models have been implemented with the *zimpl* modeling language [25] and solved using *Cplex 9.1* [21]. The tabu search heuristic was coded in C++ in the *Microsoft Visual C++* environment. To generate the instances and analyze the model results, a *Microsoft Windows* application was developed for carrying out tasks such as managing the team and distance matrix data, writing the *zimpl* file, executing *Cplex*, reading the results and displaying the schedule and trips on a graphical interface (see Figure 3).

In the non-mirrored case, schedules obtained in just a few minutes by the tabu search were on average 3% better (although not necessarily optimal) than the best solutions obtained by *Cplex* after up to 10 hours of running time. Unfortunately, the dual bounds obtained by *Cplex* are very poor so we are not able to provide meaningful optimality guarantees for the feasible schedules.

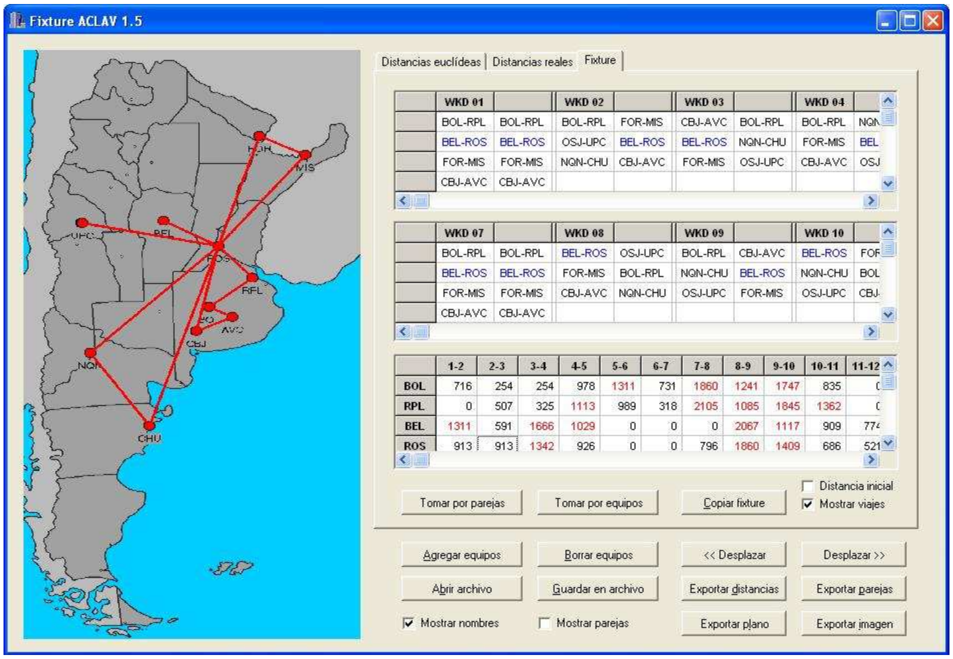


Figure 3: Computational application for managing the teams, the distance matrix and the generated schedules.

4 Results

In this section we present the results of the final schedules for the 2007-2008, 2008-2009, 2009-2010 and 2010-2011 seasons and a comparison between the manually-designed and model-generated schedules for the 2005-2006 and 2006-2007 seasons. In 2005-2006 and 2006-2007 the manually-designed schedules were implemented whereas the model-generated schedules were used from 2007-2008 to the present. The 2007-2008, 2008-2009 and 2010-2011 schedules are mirrored, optimal, and were produced by the integer programming model; the 2009-2010 schedule is non-mirrored and was obtained using the tabu search metaheuristic.

Figure 4 summarizes the results of the schedule solutions for each season. It should be kept in mind that the set of teams varied from one year to the next because of relegations and promotions to and from the Second Division. For 2005-2006 the coupling proposed by the model is different from the manually-designed couples, but for 2006-2007 the model and manual couples coincide. The model-based schedules achieve a 22.34 percent reduction in total travel distance compared to the manual one for 2005-2006 and a corresponding 15.41

percent reduction for 2006-2007, a global saving for the teams of close to US $ 60,000 in annual travel costs. Even more importantly, the reduction in travel distances would give the players more resting time before the matches. No comparisons for the years following 2006-2007 can be made since manually-designed schedules after that season were no longer developed.

A result that requires further explanation relates to 2008-2009, when the most traveled team logged a much greater total distance than the most traveled teams in the other seasons. This was due to a combination of the team's location in the far north of the country and the set of three home-away conditions it requested, which strongly affected its trip pattern. A similar situation holds for the 2010-2011 league.

The solution times for the integer programming model range from one minute to six hours on a PC with an Intel Core 2 Duo CPU running at 2.1 GHz and 2 GB of RAM. As mentioned previously, running times tend to be smaller when there are more home-away constraints (specifying that certain couples must play either home or away matches on certain weekends), which are precisely the most difficult cases to schedule manually. Execution time for the tabu heuristic typically varies between one and eight minutes, the best solutions usually found within the first minute although in such cases there is no guarantee of optimality.

The schedules generated by our techniques achieved a much smaller distance gap between the most traveled and the least traveled team, even though until the 2008-2009 league the objective function only minimized the total travel distance.

An important benefit of our computational tool is the possibility of generating different scenarios in the days prior to the schedule announcement, allowing for extensive testing with many different team couplings or even with different lists of teams (if the set of teams is not yet finalized). This last feature is significant as in most years some teams do not confirm their participation until just before the deadline. In such cases, efficient generation of schedules is crucial. The ability to determine whether all of the conditions requested by the teams can be incorporated into a feasible schedule is also considered to be a valuable feature by the competition manager.

The mathematical programming approach presented in this study has been successfully applied for the last four years, and the schedules generated for all four seasons were used by the

		Manual schedule		
Season	Teams	Total distance	Max. travel	Min. travel
2005-2006	12	135,677	15,441	9,089
2006-2007	12	135,299	13,282	9,574

		Model-based schedule		
Season	Teams	Total distance	Max. travel	Min. travel
2005-2006	12	105,356	10,723	7,670
2006-2007	12	114,445	11,017	8,333
2007-2008	12	135,043	12,702	9,649
2008-2009	11	119,245	15,284	8,541
2009-2010	11	123,244	12,938	9,535
2010-2011	12	150,334	15,770	10,442

Figure 4: Results of the manually-designed and model-generated schedules. For each season the number of teams, the total travel distance in kilometers and the travel distances of the “most traveled” and “least traveled” teams are indicated.

league. All the proposed constraints were satisfied in the cases were this was mathematically possible. Where some constraint sets turned out to be mutually contradictory the integer programming model helped to identify them, and they were then relaxed or dropped entirely in coordination with the competition manager.

Commenting on this collaborative effort with ACLAV, Association president Leonardo Carod stated that "The results were very much appreciated, especially considering the rapid solutions and the various proposals submitted for our analysis. We are extremely satisfied and expect to continue using this mathematical system in cooperation with the University and their research team."

As for future research, the model as presented here could be enhanced to take into account accommodation costs as well as travel distances in order to optimize total costs instead of just travel times.

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Appendix 1: The models

In this appendix we review the integer programming models employed in the scheduling process. We begin with a description of the model used to design the team couples and then describe a straightforward model for designing the schedule.

Integer programming model for designing the team couples

The optimal-tour coupling problem introduced in Section 3.1 is modeled as follows. Let $P = \{1, \ldots, n\}$ be the set of teams. We assume that the travel distances between the teams satisfy the triangle inequality. For ease of exposition we assume n to be even, though a similar model could also be given for an odd n. By the observations in Section 3.1, there exists an optimal solution such that the set of tours for each couple consists of a certain number of two-weekend trips and at most one single-weekend trip (or exactly one if the number of couples is odd). For example, if $n = 12$ then the optimal solution involves three two-weekend trips for each couple, one of them including the intra-couple match and thus consisting of just three matches.

To specify the model's objective function, we must explicitly represent a set of tours for each couple. We therefore assume that the matches take place in certain time slots. A set of

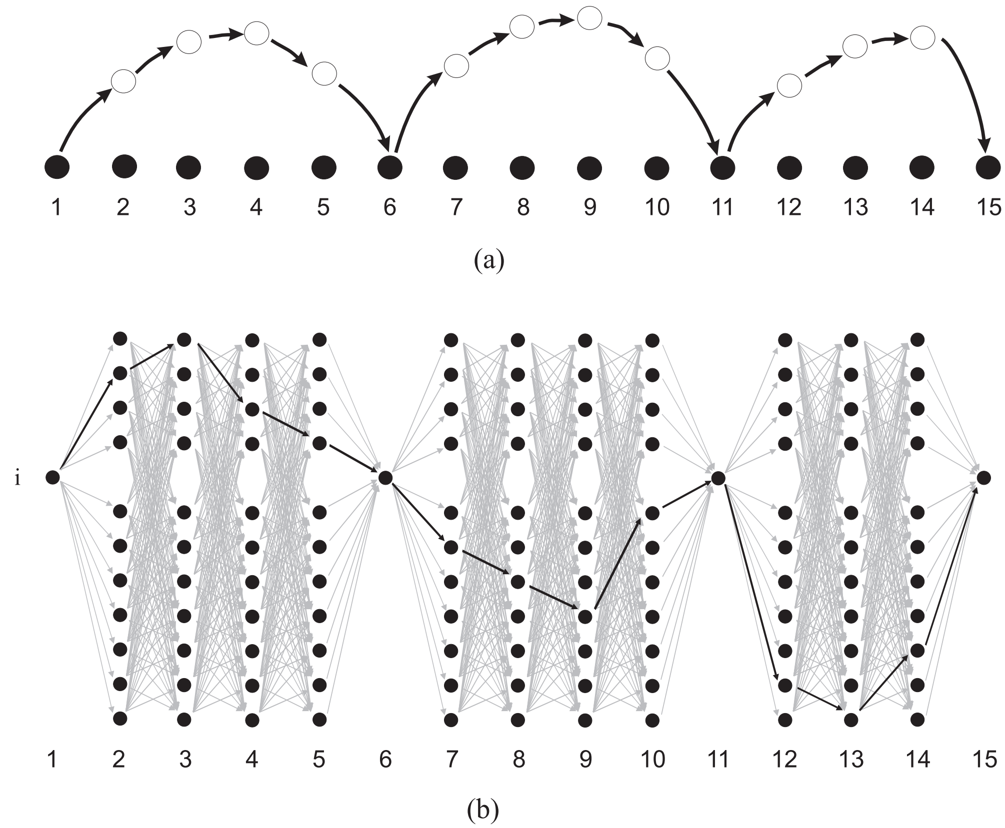

Figure 5: Representation of trips in the integer programming model for designing team couples. Figure (a) shows the $m = 15$ time slots for $n = 12$ teams and the trips made by one team. Figure (b) depicts the trip graph for $n = 12$ and team i, with a possible feasible path through the graph indicated by the black edges.

"away time slots" is defined for the away matches and a set of "home time slots" for sets of consecutive home matches. As an example, Figure 5(a) depicts the tours through the time slots for a single team in the $n = 12$ case. Slots 2 to 5 represent a two-weekend tour followed by a sequence of home matches in slot 6 (note that we do not explicitly model these home matches). Slots 7 to 10 represent a second two-weekend tour and slots 12 to 14 represent the last three-match trip including the intra-couple match as the last away match. Since, on the first trip, two other couples are visited, we must impose that in slots 2 and 3 (resp. 4 and 5) the two teams from each couple visit the two teams from the same couple.

Such a representation for the tours performed by a team does not introduce an additional constraint to the formulation (hence does not restrict the optimal solution) as, e.g., in a

12-team setting any optimal tour is composed by two four-match tours and one three-match tour. The same representation of a set of tours is designed for all of the teams, each of which will be assigned trips according to this structure. The time slots in the model do not correspond to specific dates throughout the tournament (i.e., the indices of the time slots do not correspond to the weekend indices); rather, they represent only the away matches and *sequences of home matches*. Furthermore, since in this problem we are not seeking a feasible schedule but minimizing the lower bound on tour costs instead, there are no constraints forcing a team to remain home when some other team visits its venue. On the contrary, in this representation all teams play away in time slots 2–5, 7–10, etc., and return home for time slots 6, 11, etc.

Let $T = \{1, \ldots, m\}$ be the set of time slots, where $m = n + \lceil (n-1)/4 \rceil$. By the previous description, we assume that every team plays home matches in slots $k \in H$, where $H = \{k : k = \min(m, 5t+1) \text{ for } t = 0, \ldots, n/4,\ t \in \mathbb{Z}\}$. We define $W = \{k : k = 5t+2,\ k = 5t+4 \text{ for } t = 0, \ldots, \lfloor (n-1)/4 \rfloor,\ t \in \mathbb{Z}\}$ in such a way that, for each $k \in W$, slots k and $k+1$ represent the two matches on a weekend. With these definitions, each couple must play against some other couple in slots k and $k+1$, for $k \in W$.

For $i, j \in P$, $i < j$, we introduce the binary *coupling variable* w_{ij} such that $w_{ij} = 1$ if and only if i and j belong to the same couple. For notational convenience we assume $w_{ji} = w_{ij}$ for $i, j \in P$, $i < j$. For $i, j, k \in P$ and $t \in T$, we introduce the binary *trip variable* y_{ijkt} such that $y_{ijkt} = 1$ if and only if team i travels from j to k after slot t. For each team $i \in P$, the trips made by team i can be interpreted as a path in a layered graph (see Figure 5(b)) in which the edges are the variables y_{ijkt} for $j, k \in P$ and $t \in T$. This will motivate the inclusion of flow constraints in the model. Using these variables, the integer programming model is as follows:

$$\min \sum_{i \in P} \sum_{j \in P} \sum_{k \in P} \sum_{t \in T} d_{jk}\, y_{ijkt}$$

$$\sum_{j \in P, i < j} w_{ij} \quad = \quad 1 \qquad \forall\, i \in P \tag{1}$$

$$
\begin{aligned}
y_{ijk1} &= 0 \qquad \forall\, i,j,k \in P, i \neq j && (2)\\
y_{ijkm} &= 0 \qquad \forall\, i,j,k \in P, i \neq k && (3)\\
\sum_{k\in P} y_{ikjt} &= \sum_{k\in P} y_{ijk,t+1} \qquad \forall\, i,j \in P,\ \forall\, t \in T\backslash\{m\} && (4)\\
\sum_{k\in P} y_{ikit} &= 1 \qquad \forall\, i \in P,\ \forall\, t \in H && (5)\\
\sum_{k\in P}\sum_{t\in T} y_{ikjt} &= 1 \qquad \forall\, i,j \in P, i \neq j && (6)\\
y_{ijkt} &\leq w_{jk} \qquad \forall\, i,j,k \in P,\ \forall\, t \in W,\ j<k && (7)\\
y_{ikpt} + w_{ij} - 1 &\leq y_{jpkt} \qquad \forall\, i,j,k,p \in P,\ \forall\, t \in W\ i<j && (8)\\
y_{ijkt} &\in \{0,1\} \qquad \forall\, i,j,k \in P,\ \forall\, t \in T && (9)\\
w_{ij} &\in \{0,1\} \qquad \forall\, i,j \in P,\ i<j && (10)
\end{aligned}
$$

Constraints (1) impose a matching among the teams. Constraints (2)-(3) ensure that each team starts and ends a trip at home. Constraints (4) are the usual flow conservation constraints. Constraints (5) ensures that each team returns home after every 4-match trip, i.e., for every time slot in H. Constraints (6) require each team to visit each other team exactly once. Constraint (7) asserts for every $i \in P$ that on each weekend team i must visit two teams from a single couple, and constraint (8) requires that the team coupled with i must visit the same two teams but in the opposite order. Finally, constraints (9) and (10) force the y- and w-variables to be binary. For 12 teams the model has 24,336 binary variables, 84,960 constraints and 339,156 nonzero elements.

Integer programming model for scheduling the matches

Let $C = \{1,\ldots,\lceil n/2\rceil\}$ be the set of couples (considering the uncoupled team, if there is one, as a "couple") and let $W = \{1,\ldots,2\lceil n/2\rceil\}$ be the set of weekends. For $i,j \in C$ and $k \in W$ we introduce the binary *match variable* x_{ijk} such that $x_{ijk} = 1$ if couple i plays at home against couple j on weekend k, otherwise $x_{ijk} = 0$. For $i \in C$ and $k \in W$, the variable $x_{iik} = 1$ represents the intra-couple match for couple i. For $i,j,h \in C$ and $k \in W \cup \{0\}$, we introduce the binary *trip variable* z_{ijhk} such that $z_{ijhk} = 1$ only if couple i plays away at couple j on weekend k and then away at couple h on weekend $k+1$. If couple i stays at home on weekends

k and $k+1$, then we set $z_{iiik} = 1$. We assume that each team returns home before and after each intra-couple match. This assumption is reasonable since the intra-couple distances are usually small.

For $i \in C$, assume that couple i is composed of teams $i_A \in P$ and $i_B \in P$, with $i_A \neq i_B$ and i_A being the A-team of the couple. We denote the intra-couple weekends $k_1 \in W$ and $k_2 \in W$ and assume that $k_1 + 1 < k_2$ (i.e., these weekends are not consecutive). For $i \in C$ and $k \in W$, the distance v_i^A (resp. v_i^B) traveled by i_A (resp. i_B) on the weekend k is given by:

$$v_{ik}^A = \begin{cases} \sum_{j \in C} (d_{i_A j_B} + d_{j_B j_A}) x_{jik} & \text{if } k = 1 \neq k_1 \\ \sum_{j \in C} \sum_{h \neq i} (d_{j_A h_B} + d_{h_B h_A}) z_{ijh,k-1} + \sum_{j \in C} d_{j_A i_A} z_{iji,k-1} & \text{if } k \neq 1, k_1, k_2 \\ d_{i_A i_B} + d_{i_B i_A} & \text{if } k = k_1 = 1 \\ \sum_{j \in C} d_{j_A i_A} z_{iji,k-1} + (d_{i_A i_B} + d_{i_B i_A}) & \text{if } k = k_1 \neq 1 \\ \sum_{j \neq i} d_{j_A i_A} z_{iji,k-1} & \text{if } k = k_2 \end{cases}$$

$$v_{ik}^B = \begin{cases} \sum_{j \in C} (d_{i_B j_A} + d_{j_A j_B}) x_{jik} & \text{if } k = 1 \neq k_1 \\ \sum_{j \in C} \sum_{h \neq i} (d_{j_B h_A} + d_{h_A h_B}) z_{ijh,k-1} + \sum_{j \in C} d_{j_B i_B} z_{iji,k-1} & \text{if } k \neq 1, k_1, k_2 \\ 0 & \text{if } k = k_1 = 1 \\ \sum_{j \in C} d_{j_B i_B} z_{iji,k-1} & \text{if } k = k_1 \neq 1 \\ \sum_{j \neq C} d_{j_B i_B} z_{iji,k-1} + (d_{i_B i_A} + d_{i_A i_B}) & \text{if } k = k_2 \end{cases}$$

Let k be a weekend which is not an intra-couple weekend. If $k = 1$ then the distance traveled by i_A on weekend k is $d_{i_A j_B} + d_{j_B j_A}$ if couple i plays away against couple j (given that i_A travels from home to the B-team of couple j, i.e., j_B, and then from j_B to the A-team of couple j, i.e., j_A), and is 0 if couple i stays at home. On the other hand, if $k > 1$ then we cannot assume that i_A starts its trip from its home city as i_A may have played an away match on weekend $k-1$. This implies that for $k > 1$, the distance traveled by i_A is $d_{j_A h_B} + d_{h_B h_A}$ if couple i played couple j on weekend $k-1$ and couple h on weekend k (i.e., if $z_{ijh,k-1} = 1$), and $d_{j_A i_A}$ if couple i played couple j on weekend $k-1$ and returned home for weekend k (i.e., if $z_{iji,k-1} = 1$).

Consider now the first intra-couple weekend k_1 (recall that i_A plays away at i_B on the weekend k_1 and i_A plays home against i_B on the weekend k_2). If $k_1 = 1$ then i_A travels to i_B

(to play i_B) and returns home, hence the trip distance at k_1 is $d_{i_A i_B} + d_{i_B i_A}$ (and i_B stays at home, so $v^B_{ik_1} = 0$ in this case). On the other hand, if $k_1 > 1$ then we must also consider the trip from j_A back to i_A (since we assume every team returns home before and after each intra-couple match) if couple i played an away weekend against couple j on weekend $k-1$ (i.e., if $z_{iji,k-1} = 1$). Note that if couple i played at home on weekend $k-1$, then $z_{iii,k-1} = 1$ and no additional term is added. For the second intra-couple weekend k_2, only the trip back home must be considered for team i_A as it stays home for a match against i_B. A symmetrical analysis holds for the definition of v^B_{ik}.

Note that the definitions of v^A_{ik} and v^B_{ik} involve linear expressions in the x- and z-variables and can therefore be incorporated into the integer programming model. With these definitions, the following integer programming model represents the problem of scheduling the matches with the minimum total travel distance. The other objective functions discussed previously can be modeled similarly.

$$\min \sum_{i \in C} \sum_{k \in W} v^A_{ik} + v^B_{ik}$$

$$\sum_{k \leq |W|/2} (x_{ijk} + x_{jik}) = 1 \qquad \forall i, j \in C, i \neq j \tag{11}$$

$$x_{iik} = 1 \qquad \forall i \in C, k = k_1, k_2 \tag{12}$$

$$x_{iik} = 0 \qquad \forall i \in C, k \in W \backslash \{k_1, k_2\} \tag{13}$$

$$x_{ijk} = 0 \qquad \forall i \in C, i \neq j, k = k_1, k_2 \tag{14}$$

$$x_{ij,k+|W|/2} = x_{jik} \qquad \forall i, j \in C, i \neq j, k \leq |W|/2 \tag{15}$$

$$\sum_{j \neq i} \sum_{t=0}^{2} x_{ij,k+t} \leq 2 \qquad \forall i \in C, \forall k \in W : \{k, k+1, k+2\} \subseteq W \backslash \{k_1, k_2\} \tag{16}$$

$$\sum_{j \neq i} \sum_{t=0}^{2} x_{ji,k+t} \leq 2 \qquad \forall i \in C, \forall k \in W : \{k, k+1, k+2\} \subseteq W \backslash \{k_1, k_2\} \tag{17}$$

$$z_{ijhk} \geq x_{jik} + x_{hi,k+1} - 1 \qquad \forall i, j, h \in C, \forall k \in W \tag{18}$$

$$z_{iihk} \geq x_{ijk} + x_{hi,k+1} - 1 \qquad \forall i, j, h \in C, \forall k \in W \tag{19}$$

$$z_{ijik} \geq x_{jik} + x_{ih,k+1} - 1 \qquad \forall i,j,h \in C, \forall k \in W \tag{20}$$

$$z_{iiik} \geq x_{ijk} + x_{ih,k+1} - 1 \qquad \forall i,j,h \in C, \forall k \in W \tag{21}$$

$$x_{ijk} \in \{0,1\} \qquad \forall i,j \in C, \forall k \in W \tag{22}$$

$$z_{ijhk} \in \{0,1\} \qquad \forall i,j,h \in C, \forall k \in W \tag{23}$$

Constraints (11) ensure that each match is played exactly once in the first half. Constraints (12)-(14) impose that intra-couple matches are played on weekends k_1 and k_2 only. Constraints (15) were used with the mirrored schedules and were dropped for the 2009-2010 season. Constraints (16) (resp. (17)) ensure that no couple plays more than two home (resp. away) weekends, and constraints (18)-(21) define the trip variables in terms of the match variables. Note that constraints (19)-(21) can be equivalently replaced by a smaller set of constraints by aggregating over the teams j and h (where suitable). In our experience with the Argentine volleyball league instances, such replacement does not seem to affect the *Cplex* solution times in a significative way. Finally, constraints (22)-(23) force the variables to be binary. For 6 team couples the model consists of 3,168 binary variables, 10,104 constraints and 35,098 nonzero elements.

Appendix 2: The tabu search heuristic

In this appendix we describe the implementation of a tabu search heuristic for the TTP, taking into account the particular features of the Argentine volleyball league.

The search space is given by all of the solutions that satisfy all the constraints except those requiring that every team play no more than two consecutive home (resp. away) weekends. Whenever a team violates this constraint a penalty is added to the objective function. Our experiments suggest that such a search space is more effective than one involving only the feasible solutions. We construct initial feasible solutions following the procedure proposed in [12]. This procedure starts with an empty schedule and randomly adds matches to this schedule, taking care of avoiding repeated matches.

We utilize the following neighborhoods throughout the search process, allocating different amounts of running time to each one according to their effectiveness:

- *Partial weekend exchange.* This neighborhood consists in picking two couples c_1, c_2 and two weekends w_1, w_2 such that there exist two couples c_3 and c_4 satisfying that c_1 plays c_3 (resp. c_4) on w_1 (resp. w_2) and c_2 plays c_4 (resp. c_3) on w_1 (resp. w_2). We swap the matches of c_1 and c_2 on weekends w_1 and w_2 so that c_1 plays c_4 (resp. c_3) on w_1 (resp. w_2) and c_2 plays c_3 (resp. c_4) on w_1 (resp. w_2), and consider all 2^4 combinations of home-away status for these four matches. This neighborhood turned out to be the most effective one, and is thus used in most of the search process.

- *Weekend exchange.* This neighborhood consists of all the schedules obtained by exchanging all the matches on weekend w_1 with all the matches on weekend w_2 for any $w_1 \neq w_2$. Note that this neighborhood is composed of $O(n^2)$ solutions.

- *Couple exchange.* This neighborhood consists of all the schedules obtained by exchanging couples c_1 and c_2 for any $c_1 \neq c_2$, i.e., if couple c_1 (resp. c_2) plays $c'_1(k)$ (resp. $c'_2(k)$) on weekend k, then in the neighborhood couple c_1 (resp. c_2) plays $c'_2(k)$ (resp. $c'_1(k)$) on weekend k for every weekend k.

- *Home/away exchange.* This neighborhood consists of all the schedules obtained by exchanging the home-away condition of the two matches between couples c_1 and c_2 for any $c_1 \neq c_2$.

The tabu list is managed with a standard FIFO procedure and includes the last $t = 8$ movements performed on the incumbent solution. The *intensification strategy* affects the search process by randomly selecting values of t from $\{4, 5, 6, 7\}$ every 15,000 iterations. This strategy turned out to be very effective in our experiments. On the other hand, the *diversification strategy*, which is applied after a certain number of iterations with no improvements in the best solution, consists in performing several consecutive partial weekend exchanges in order to construct feasible solutions with major differences from the current solution.